INTELLECTUAL ALGEBRA,

BEING

ORAL EXERCISES

IN

ALGEBRA;

FOR

COMMON SCHOOLS.

By DAVID B. TOWER, A.M.,

Late Principal of the Eliot Grammar School, Boston, and of the Penn Institute for the Instruction of the Blind; author of "The Gradual Reader, or Exercises in Articulation."

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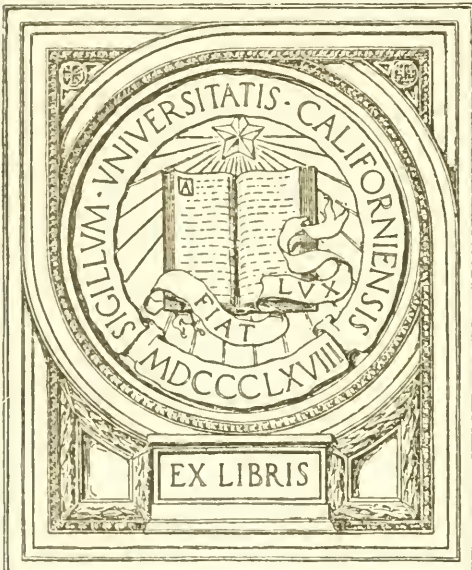

INTELLECTUAL ALGEBRA;

OR,

ORAL EXERCISES

IN

ALGEBRA;

FOR

COMMON SCHOOLS,

IN WHICH

ALL THE OPERATIONS ARE LIMITED TO SUCH SMALL NUMBERS AS NOT TO EMBARRASS THE REASONING POWERS, BUT, ON THE INDUCTIVE PLAN, TO LEAD THE PUPIL UNDERSTANDINGLY, STEP BY STEP, TO HIGHER MENTAL EFFORTS:

ADAPTED

TO PREPARE THE PUPIL FOR THE STUDY OF WRITTEN ARITHMETIC,

AND DESIGNED TO BE

INTRODUCTORY TO HIGHER TREATISES ON ALGEBRA

BY

DAVID B. TOWER, A. M.,

LATE PRINCIPAL OF THE ELIOT GRAMMAR SCHOOL, BOSTON, AND OF THE PENN. INSTITUTE FOR THE INSTRUCTION OF THE BLIND; AUTHOR OF "THE GRADUAL READER, OR EXERCISES IN ARTICULATION."

"Divide and subdivide a difficult process until your steps are so short that the pupil can easily take them."—*Abbott's Teacher.*

THIRTEENTH EDITION.

NEW YORK:

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1855.

PREFACE.

It is now three years and a half since this work was prepared for the use of the *blind* under the author's charge; and it took this form from the necessity for *oral* instruction, in their peculiar case. The great advantages derived by them from these exercises, in developing and strengthening the mental powers, in fixing the attention, and in awakening a strong desire to acquire knowledge *understandingly*, by seeking the *why* and *wherefore* at every step in their progress, wrought in the author a firm conviction that algebra, in this shape, should *precede* written arithmetic; and that such would be the case at no distant period. About three years ago, the design of the author to publish a mental algebra, was communicated to two of the most distinguished teachers in this city, and met with their approval; but unceasing duties in managing a large institution, have hitherto delayed its publication. It is now printed for the use of the private pupils under the author's care; and, with the hope that it may be as successful with the *seeing*, as it has been with the *blind*, it is humbly offered to the public. Should it succeed in making algebra a common school study, and should it do for that study, in some small degree, what "Colburn's First Lessons" have done for arithmetic, the author will congratulate himself that one original idea of his has been of value to the young.

These exercises gradually lead the pupil, step by step, from the simplest to more complicated reasoning; teaching only *one thing at a time*, and rendering that one thing

familiar, before the attention is called to another. The additional strength that the mind daily gains by such repeated exercise, can hardly be conceived but by the experienced teacher. The increase of intellectual power from such a source, almost equals the accession of physical strength, which ancient fable tells us a man acquired, by carrying a calf daily till it grew to be an ox.

Nor will an algebraic process of reasoning, however long, seem at all difficult for the memory, where the numbers are small, when it is recollected that you have to stand on but one round of a ladder to reach the next higher round; and that this process, continued, easily carries you to the top. The last step requires no greater effort than the first. So in an algebraic solution, where only one symbol is used to express the conditions of a question, *one step only* need be held in mind to reach the next, and it need be held only till the next *is* reached. Each successive step is dependent on the preceding, and is derived from it by a process of reasoning generally limited to that step. Even in using several symbols, the mind is easily trained to discriminate and retain whatever is needed as an argument in the solution, and to lay aside at once all the steps of the process by which that conclusion was reached.

The author found that his *blind* pupils, thus taught, gained intellectual strength sufficient to solve, mentally, questions requiring five different letters and equations to express the conditions. That part of the manuscript, however, has been omitted; and all such questions have been carefully excluded, as not coming within the design of this elementary treatise.

Furthermore, an algebraic solution is far less mechanical than an arithmetical one is often permitted to be. There is no remembering abstract numbers, to undergo operations prescribed by rule; but the reasoning on each successive step attaches a *meaning* to it, dependent on the connection

between the several parts of an equation. Thus, the pupil is delighted to exercise his powers on an equation; it is a conflict which excites his mental energy; and who remembers not his boyish satisfaction in surmounting a difficulty? There is a peculiar pleasure in this study, when rightly presented to the young, which seldom fails to interest and rouse the pupil, though no other study has been able to call forth any vigorous effort. Curiosity, in youth, the main spring of intellect, is hereby made to act in its proper sphere; the kind interest, the skill, and the superior intelligence of the teacher, must direct, while this curiosity needs a guide; but, once on the track, with such a motive power, the wheels can never cease to revolve.

Every teacher knows, from experience, how readily a pupil will understand an arithmetical question, and with what facility he will reason upon it, when *small* numbers are substituted for *large* ones, without altering a single condition of the question, however difficult and unintelligible it appeared before. Large numbers embarrass the pupil; and he should learn to reason with small numbers at first, till he gradually acquires strength to wield larger ones. On this *principle* the author has *based* this work. The numbers are *small*, that the pupil may solve the questions *mentally*. Although intended solely as *oral exercises*, the teacher will perceive that the questions may be solved on the slate, and that *written* algebra can be taught from *this book* as well as from a larger treatise.

A Key, containing answers, solutions, and suggestions for teachers, is in press, and will be of assistance, especially to those who have neither taught nor studied algebra.

The author invariably required his pupils to make questions in each successive section; thus he ascertained that each principle was clearly understood before he proceeded to the next. This would be found very useful, and might be made a *home* exercise.

To his brethren the Boston Teachers this work is respectfully dedicated, by their friend and former associate, with the earnest desire that their efforts in the cause of education, and their devotion to the interests of the young may be duly appreciated and rewarded

D. B. P.

No. 13, Somerset Street,
Boston, April 8, 1835.

SUGGESTIONS TO TEACHERS.

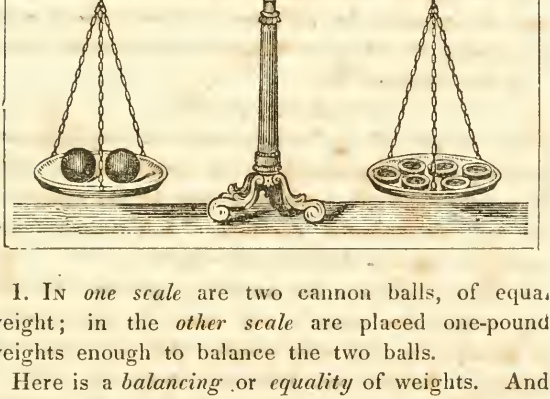
After a question is read or given out, call on some *one* of the class to repeat it; on *another*, to state what is required; on a *third*, for the data, or known conditions on which the question is based, and from which the answer is to be deduced; on a *fourth*, to state what x, or any other symbol used, is to represent in the given case; on a *fifth*, to use a symbol or symbols in accordance with the expressed conditions; on a *sixth*, to make an equation from the materials, using the symbols to represent the unknown quantities; on a *seventh*, to prove the equation, thus made, to be true, stating *why* and *wherefore;* on an *eighth*, to give the *first* step in reducing this equation, with the reasons for it; on a *ninth*, for the *next* step; and so on, till the value of the symbol or symbols used, is found *Another* pupil should then be required to prove the whole, by using the numerical value thus found in the several conditions of the question. When there are *several* ways of reducing an equation, *other* pupils can go through with each of them in the same manner. By calling on the pupils promiscuously, the attention of all is thus confined to each step of the process, and the greatest benefit is secured. By this method, in a class of forty, each pupil does something in the way of recitation, towards the solution of every third or fourth question, and silently is compelled to attend to the whole process of all.

In addition to the *oral* lesson of the day, thus recited, the class may be required to have, on their slates, the lesson of the *preceding* day. Here, too, each step of the solution should be explained in a similar manner by the class. This will serve for a review, and, at the same time, teach *written* Algebra.

The Key will be found of great assistance to the teacher in hearing a class, interrupted, as he constantly is, by the many who demand his care and attention Besides, it will be useful for assistants and monitors.

INTELLECTUAL ALGEBRA.

SECTION I.



1. In *one scale* are two cannon balls, of equal weight; in the *other scale* are placed one-pound weights enough to balance the two balls.

Here is a *balancing* or *equality* of weights. And since it takes six one-pound weights to balance the two balls, two balls weigh six pounds; and the expression,

Two balls are equal to six pounds,

is an *equation*. This equation may, by using = the *sign* of *equality*, be expressed thus:

2 balls = 6 pounds.

2. In the equation

$$2 \text{ balls} = 6 \text{ pounds},$$

the number of pounds needed to *balance one* ball, or the *weight* of *one* ball, is the *unknown quantity* to be found out or *determined.*

If six pounds balance two balls, it is evident, that *one half* as many pounds will balance *one* ball; or, if two balls weigh six pounds, *one* ball will weigh *one half* of six pounds, which is three pounds; because, if *one* ball weighs *three* pounds, *two* balls will weigh *two times* three pounds, which is six pounds.

3. In algebra, some *symbol,* as the letter x, or y, is used to represent the *unknown* or *undetermined number;* that is, the *thing,* or *things,* required to be found.

In the equation

$$2 \text{ balls} = 6 \text{ pounds},$$

if the *weight* of *one* ball is required, the *unknown* quantity or *thing* sought, is the *weight* of *one* ball. If a *symbol,* as the letter x, is used for the *weight* of *one* ball, the *unknown quantity,* x, will *represent* the *number* of pounds that *one* ball weighs.

4. If x represents the *number of pounds* that one ball weighs, *two times* x will *stand for* the number of pounds the two balls weigh; that is, if one ball weighs x pounds, two balls will weigh two times x pounds, which may be expressed thus: $2x$ *pounds.* Since two balls weigh six pounds, $2x$ pounds, *representing* the weight of two balls, must be *equal to* six pounds. We have, then, this equation,

$$2x \text{ pounds} = 6 \text{ pounds},$$

and $2x$ is *one member* of the equation, and 6 is the

other member. Now, we wish to find the *value* of x, or the *number* that it *represents*.

5. In the equation

$$2x \text{ pounds} = 6 \text{ pounds},$$

one x, which is *one half* of two x, must be equal to *one half* of six pounds, which is three pounds. Then *three* is the *number* represented by x, and the *value* of x is now *known* or *determined* to be three. The *weight* of *one* ball is therefore *three pounds;* and *one* ball in *one* scale will balance *three* pounds in the *other*.

6. Since *one ball* in *one* scale balances *three pounds* in the *other;* if *one more* ball, of the same weight, be put into the scale with the first ball, it is evident that *three one-pound weights* must be added to the three pounds already in the other scale, that the *balance* or *equality* may still be preserved. If a *third* ball be put with the *two* balls, *three more* pounds must be put with the *six* pounds, that the *balance* or *equation* of weight may still exist.

7. Therefore, if *equal weights* be *added* to each scale, when the scales *are balanced*, the *balance* or *equality* continues. We also see, that if *one ball* = *three pounds* in weight, *two times* one ball, that is, two balls, = *two times* three pounds, that is, six pounds; and *three times* one ball, or three balls, = *three times* three pounds, or nine pounds; so that, if *equal weights* be equally increased, the *balance* or *equality* between them still exists.

8. In the equation

$$x = 3,$$

if x be added to x, *the first member*, it is evident that 3, the value of x, or number it represents, must be

added to 3, the *second member*, that the *equality* may be preserved. Then

x added to x will be equal to 3 *added to* 3;

or, *x and* $x = 3$ *and* 3;

or, using *plus*, $+$, the *sign* of *addition*

$$x + x = 3 + 3.$$

But $x + x = 2x$, and $3 + 3 = 6$; therefore, the *equation* now is,

$$2x = 6.$$

If x be *again* added to the first member, and the *number* it *represents* be added *again* to the second member, the equation will be

$$x + 2x = 3 + 6,$$

or, $3x = 9$.

9. Therefore, if *equal quantities* be *added* to *each member* of an *equation*, the *equality* still continues. We also see that if $x = 3$,

twice $x =$ *twice* 3,

and using $\times$, the *sign* of *multiplication*,

$$2 \times x = 2 \times 3;$$

so that, if *each member* of an *equation* be *multiplied* by the *same number*, the *equation* or *equality* will still be preserved.

10. Two balls in one scale balance six pounds in the other, and each ball weighs three pounds. If *one* of the two balls be taken out of the scale, it is evident that *its* weight, or *three one-pound* weights, must be taken from the other scale, that the *balance* or *equality* of weights may still continue. Then the equation will be, *two balls* with *one ball* taken from them $= 6$ *pounds* with 3 *pounds* taken from them.

11. Therefore, if *equal weights* are *taken from* each

scale when the scales are balanced, the *balance* or *equality* continues. We see also, that if *two balls* = *six pounds* in weight, one half of two balls, which is one ball, is equal in weight to one half of six pounds, which is three pounds; or, 2 balls divided by 2 = 6 pounds divided by 2; that is, if *equal weights* are equally diminished, or if the *same part* of equal weights is taken away, the *balance* or *equality* between the remaining parts still exists.

12. If x be taken from $2x$, the *first member* of the *equation*,

$$2x = 6,$$

it is evident that 3, the *value* of x, or the *number* that it *represents*, must be taken from 6, the *second member*, that the *equality* may be preserved. Then $2x$ with x taken from them = 6 with 3 taken from them; that is,

$$2x \text{ less } x = 6 \text{ less } 3;$$

or, using —, the *sign* of *subtraction*, called *minus*, or *less*,

$$2x - x = 6 - 3.$$

But $2x - x = x$; and $6 - 3 = 3$; and the *equation* now is,

$$x = 3.$$

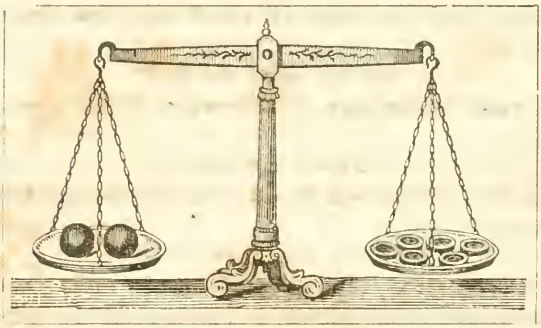
13. Therefore, if *equal quantities* be *taken from each member* of an *equation*, the *equality* still continues between the remaining parts of each member. We also see, that if $2x = 6$; $2x$ divided by 2, that is, using a *sign* of *division*, $\frac{2x}{2}$ = 6 divided by 2, that is, $\frac{6}{2}$; or that one half of $2x$ = one half of 6.

So that, if *each member* of an *equation* be *divided*

by the *same number*, the *equality* or *equation* will still be preserved.

Remark. — Such questions should be asked as are necessary to ascertain the accuracy and clearness of the pupil's knowledge of this section. This each teacher will do for himself better than the author can do it for him. Printed questions, for such purposes, in the hands of a pupil, too often serve but as moulds in which to run his answers; preventing, rather than aiding, mental effort.

SECTION II.



1. If two cannon balls, of equal weight, in one scale, are balanced by six one-pound weights placed in the other scale, how many of these one-pound weights will it take to balance one ball? or, what is the weight of one ball?

Explanation and Solution.

Let x represent the *answer sought*, or unknown quantity;

that is, let x represent the *weight* of *one ball.*

Then, two times x pounds will stand for the weight of two balls; thus,

two balls weigh $2x$ pounds.

But, by a statement or condition of the question,

the two balls weigh six pounds.

Therefore, $2x$ pounds must be equal to six pounds.

If $2x$ pounds are equal to six pounds,

or $2x = 6$;

then one x will be equal to one half of six pounds, or, one half of $2x$, which is x, is equal to one half of six pounds, which is three pounds.

Or it may be expressed thus:

$$x = \frac{6}{2};$$

that is, x equals six divided by two.

Answer. The weight of a ball is 3 lbs.

2. One x is what part of two times x?

3. In the equation $2x = 8$, to what part of eight is one x equal?

4. If each member of the equation $2x = 8$, be divided by two, what equation will express the quotient?

5. What will represent the sum of x and x?

6. Express in one term the three terms $x + x + x$. What will represent their sum?

7. If each member of the equation $3x = 12$, be divided by 3, what equation will express the result?

8. George and Charles are to have four balls; and one is to have as many as the other. How many balls will each have?

Let x represent the number of balls that George will have.

Then x will also represent the number Charles will have.

Now, if x, or George's number of balls, be added to x, or Charles's number of balls, the sum will be $x + x = 2x$ balls.

$2x$ will represent the number of balls that both will have;

and, since both together will have 4 balls,

$2x$ balls must be equal to 4 balls.

If $2x = 4$ balls, one x will equal one half of 4 balls, which is 2 balls.

Therefore, $x = 2$ balls, and each boy will have 2 balls.

9. If four be divided into two equal parts, what will one of the parts be?

Let $x =$ one part;

then another x will represent the other part;

and $x + x$, which is $2x$, will represent both parts, or the whole number.

But the whole number is 4;

therefore $2x = 4$.

If $2x = 4$, x will equal one half of 4.

Therefore, $x = 2$,

and one of the parts is 2.

10. Mary and Anna, together, have eight books; and Mary has as many as Anna. How many books has each?

11. John and James are to have equal shares of

eighteen chestnuts. How many chestnuts will each have?

12. Two boys agree to take equal shares of twenty-four apples. How many apples may each take?

13. What number must be added to itself, that the sum may be six?

14. If x represents some number, what will represent the same number added to itself?

15. Robert has sixteen apples, which he wishes to divide equally between John and himself. How many apples will John have?

16. When a certain number is added to itself, the sum is ten. If x represents the number, what will represent the number added to itself? To what will the number added to itself be equal? What is the number?

17. In two classes there are thirty pupils; and there is the same number of pupils in each class. If x represents the number in one class, what expression will represent the number in both classes? To what must this expression, that represents the number in both, be equal? What is the number of pupils in each class?

18. What number must be added to itself, that the sum may be twenty-four?

19. Add such a number to itself, that the sum shall be sixteen. What will represent the number added to itself? To what will the number added to itself be equal? What will the number be?

20. Add such a number to itself that the sum shall be thirty. What will the number be?

21. Two boys together have twenty-two cents; and

one has as many as the other. If x represents the number of cents that one boy has, what will represent the number that both have? To what will this expression be equal? How many cents will each have?

22. What number must be added to itself, that the sum may be twenty?

23. There are eighteen chairs standing in two rows, with the same number in each row. How many chairs are there in each row?

24. Divide 12 into two equal parts. If x represents one of the parts, what will represent the other? What will represent both parts? What will both parts equal? What will one part be?

25. George is as old as John, and the sum of their ages is twenty-six years. If x be used to represent the age of John, what will represent George's age? What expression will stand for the sum of their ages? What will this expression equal? What is the age of each?

26. In the equation $2x = 28$ cents, what is the value of x?

27. Anna gave some money to a poor woman, and Josephine gave her as much as Anna did. She received from both thirty cents. How many cents did each give her?

28. If a line, fifty feet long, be cut into two equal parts, how long will one of the parts be?

29. What number must be added to itself, that the sum may be sixty?

30. The number of full barrels in a store is equal to the number of empty ones; and the sum of both is forty. How many are there of each kind?

31. In a school of forty pupils, there are as many boys as girls. How many pupils are there of each sex?

32. Twenty-four horses and cows are feeding in a pasture, of each an equal number. How many are there of each?

33. If a line, thirty-three feet long, be cut into three equal pieces, and if x represents one of the pieces, what will represent each of the other two pieces? What will represent the sum of the pieces? To what will the expression, that represents the sum of the pieces, be equal? How long will each of the pieces be?

34. John, Charles, and Caleb, have each an equal number of blocks, and together they have twenty-one. How many blocks has each?

35. Divide fifteen into three equal parts. If x represents one of the parts, what will stand for each of the other two parts? What will express the sum of the parts? What will one of the parts be?

36. A farmer wishes to put ninety sheep into three pastures, so that there may be an equal number in each. How many sheep will there be in each pasture?

37. George, Anna, and Charles, are to have equal shares of twenty-seven peaches. If x represents Anna's share, what will represent the share of each of the other two? What will represent the sum of their shares? What will the expression for the sum equal? How many peaches will each have?

38. Divide thirty-six plums among three boys, giving the same number to each. How many plums will each boy receive?

39. What number must be added once to itself that the sum may be forty?

40. What number must be added twice to itself that the sum may be eighteen?

41. Four men contributed equally to purchase a cow for a poor neighbor. The cow cost twenty-four dollars. How many dollars did each man give?

42. What number must be added twice to itself, hat the sum may be thirty?

43. Divide thirty-six into four equal parts. If x equals one part, what will be the sum of the parts? Of how many will one of the parts consist?

44. What number must be added three times to itself, that the sum may be twenty-four?

45. If $4x = 20$, what part of 20 will one x equal?

46. If a number be added four times to itself, the sum will be thirty-five. What is the number?

47. What will express the sum of $x + x + x + x$?

48. If x, and x, and x, be added, what will be the sum?

49. Express in one term, the sum of $x + x + x + x + x$.

50. How many times x are $x + x + x$? What term will express the sum?

SECTION III.

1. Two cannon balls, one weighing twice as much as the other, placed in one scale, are balanced by twelve one-pound weights in the other scale. What is the weight of each ball?

Let $x =$ the weight of the lighter ball;
then $x + x = 2x$ will be the weight of the other ball,
and $x + 2x = 3x$ will represent the weight of both balls.

Both balls, then, will weigh $3x$ pounds.

But, by a condition of the question, the two balls weigh 12 pounds.

Therefore, $3x$ pounds must be equal to 12 pounds;
expressed thus; $3x = 12$.

If $3x$ pounds are equal to 12 pounds,
x pounds, which is *one third* of $3x$ pounds,
will be equal to *one third* of 12 pounds;
therefore, $x = 4$ pounds, the weight of the lighter ball;
and $2x =$ twice 4, which is 8 pounds for the heavier ball.

Or,
dividing each member of the equation,
$3x = 12$,
by 3, gives the new equation
$x = 4$, as above.

2. If $3x = 12$, to what part of 12 will x be equal?

3. If x be added to $2x$, what term will express their sum?

4. If each member of the equation $5x = 15$, be divided by 5, what equation will represent the quotient?

5. If $x + 2x + 3x$ be united in one term, what will represent their sum?

6. Divide nine balls between George and Charles, giving to George twice as many as to Charles. How many balls will each receive?

Let x represent the *undetermined* or *unknown* number of balls which Charles is to have;

then two times x, or twice the *unknown* number, will stand for the number of balls which George is to have.

Now, x, or Charles's share, added to $2x$, or George's share, will be $3x$, or the number of balls that both will have.

But both together are to have 9 balls;

therefore, $3x$ balls must be equal to 9 balls;

and x balls will be equal to one third of 9 balls;

then $x = 3$ balls, or Charles's share.

Since x, the *unknown* number, is found to represent 3 balls, $2x$, or twice the number, will be twice 3 balls: therefore, George's share will be 6 balls

Or,

dividing the equation

$$3x = 9$$

by 3, gives the new equation

$$x = 3,$$ as above.

7. In the equation $x + 2x = 24$, what number is represented by x?

8. There are two numbers, one of which is twice the other, and their sum is fifteen. If x represents the smaller number, what will represent the larger? What will express the sum of the two numbers? To what will the sum of the two numbers be equal? What are the numbers?

9. Anna is twice as old as Mary, and the sum of their ages is eighteen years. What is the age of each?

10. The sum of two numbers is twelve, and one is twice the other. What are the numbers?

11. George has twice as many books as Thomas, and they both together have twenty-one. How many books has each?

12. Divide eighteen into two such parts, that one part shall be twice the other. If $x =$ the smaller part, what will represent the larger? What will express the sum of the parts? What will the parts be?

13. Charles and Anna have fifteen blocks for play, but Charles has twice as many as Anna. How many blocks has each?

14 Divide twenty-one into two such parts, that one part shall be twice the other. What are the parts?

15. Rollo and Lucy picked up thirty shells on a beach, and they wish to divide them so that Lucy shall have twice as many as Rollo. How many shells can each have?

16. There are two numbers, one of which is twice the other, and their sum is forty-five. What are the numbers?

17. Divide thirty-six into two such parts, that one shall be twice the other. What will each part be?

18. In a pasture there are twenty-four horses and cows feeding together, and the number of cows is double the number of horses. How many are there of each

19. The sum of two numbers is thirty-nine, and one is as large again as the other. What are the numbers?

20. What number must be added twice to itself, that the sum may be nine?

21 Find two numbers, whose sum shall be forty-

eight, and one shall be twice the other. What will the numbers be?

22. What number must be added to twice itself, that the sum may be thirty?

23. There are two numbers, one of which is three times the other, and their sum is twenty-four. If x represents the smaller number, what will be the larger? What will express their sum? What are the numbers?

24. John and William wish to divide thirty-six cents, so that John shall have three times as many as William. How many will each have?

25. What number must be added twice to itself that the sum may be twenty-one?

26. What number must be added to twice itself that the sum may be forty-eight?

27. The sum produced by adding a number twice to itself is twenty-seven. What is the number?

28. What number must be added three times to itself, that the sum may be thirty-two?

29. Robert and Mary wish to share forty peaches, so that Mary may have three times as many as Robert. How many can each have?

30. The sum of two numbers is sixty, and one is four times the other. What are the numbers?

31. If thirty-six be divided into three equal parts, what will one of the parts be?

32. What number must be added four times to itself, that the sum may be thirty-five?

33. The number of cows and sheep together in a farm-yard is seventy-five, and there are four times as many sheep as cows. How many are there of each?

34. Find two numbers whose sum will be seventy-two, and one will be five times the other. What are the numbers?

35. Divide eighty-four into two such parts, that one shall be six times the other. What will one of the parts be?

36. There are seven times as many sheep as lambs in a pasture, and in all there are ninety-six. How many are there of each?

37. There are two numbers, one of which is eight times the other, and their sum is fifty-four. What are the numbers?

38. What number must be added to nine times itself, that the sum may be one hundred and thirty?

39. A man and a boy together receive for their work fifty-five dollars; and the man's share is ten times as large as the boy's. How many dollars does each receive?

40. If x be multiplied by 3, what expression will represent the product?

41. If x be multiplied by 7, what will express the product?

42. If 7 be multiplied by x, what will express the product?

43. What is the product of 9 multiplied by x?

44. What is the product of x multiplied by 9?

45. What will represent the product of x times 5?

46. What expression will represent x times 12?

47. If $3x$ be added to $4x$, what will represent their sum?

48. What will express the sum of $2x + 3x + 4x$?

49. What one term will represent the sum of $5x+3x+x$?

50. Two men, working for a dollar a day, receive one hundred and thirty-two dollars for their labor. A worked eleven days to B's one. If x represents the number of days B worked, what will represent the days A worked? What will express the number of days both worked? If each received 1 dollar for a day's work, what will express the number of dollars he received for x days? If A received x dollars for working x days, what will express the number of dollars he received for working $11x$ days? What expression will represent the dollars both received, and to what number of dollars must this expression be equal? How many days did each work, and how many dollars did each receive?

51. The sum of two numbers is two hundred; and one is nineteen times the other. If x represents the smaller, what will stand for the larger? What are the numbers?

52. One of two numbers is twelve times the other, and their sum is thirty-nine. What are the numbers?

53. Divide one hundred and fifty dollars between two men, giving one fourteen times as many as the other. How many dollars will each receive?

54. George and Charles paid eighty cents for a sled; but George paid fifteen times as much as Charles. What did each pay?

55. A man paid one hundred and twenty dollars for a horse and saddle, and the horse cost nine times as much as the saddle. How many dollars did he pay for each?

56. What number must be added to nineteen times itself, that the sum may be one hundred?

57. Find the sum of $3x+6x+12x$.

58. What is the sum of $x+2x+4x$?

59. Unite the following terms; $x+3x+9x$.

60. Express in one term the sum of $2x+8x+x$

61. What is the sum of $5x+10x+15x$?

62. Three to be multiplied by x, by using $\times$, the *sign* of *multiplication*, may be expressed thus; $3\times x$ What will the expression be after the multiplication is performed?

SECTION IV.

1. One cannon ball and two one-pound weights in one scale, are balanced by six one-pound weights in the other scale. What is the weight of the ball?

Let x represent the weight of the ball.

Then x pounds plus 2 pounds will be the number of pounds in one scale.

Since these balance the six pounds in the other scale, x pounds and 2 pounds are equal in weight to 6 pounds; that is,

$$x+2=6.$$

Now, if 2 pounds be taken out of the scale containing the *ball and 2 pounds*, it is evident that 2 pounds must be taken out of the scale containing the 6 *one-pound weights*, that the *balance* or *equality* may be preserved.

Thus the *ball alone* in one scale balances the 4 pounds in the other.

If 2 be taken from each member of the equation,

$$x + 2 = 6,$$

the equation will be, $x + 2 - 2 = 6 - 2$, or $x = 4$.

Therefore the weight of the ball is 4 lbs.

2. If 2 be taken from the expression $x + 2$, what will be the remainder? If 2 be taken from 6, what will be the remainder?

3. If 2 be taken from each member of the equation $x + 2 = 6$, what equation will express the remainder?

4. If 5 be taken from each member of the equation $x + 5 = 13$, what equation will represent the remainder?

5. George gave two apples to Charles, and then Charles had seven. How many had Charles before he received the two from George?

6. If 4 be taken from each member of the equation $x + 4 = 9$, the subtraction may be expressed thus; $x + 4 - 4 = 9 - 4$. What will the equation be, after the terms are united?

7. Anna says, "If you give me three more books, I shall have fifteen." How many has she now?

8. A purse lacks five cents of being filled, and it will hold twenty cents. How many cents are in it?

9. A room contains twenty-four chairs, and only six are empty. How many are occupied?

10. What number is that, to which if six be added the sum will be fifteen?

11. The sum of two numbers is ten, and the

larger number is two more than the smaller. What are the numbers?

Let x represent the smaller number;

then $x+2$ will be the larger;

and $x+x+2$, or $2x+2$, will be the sum of the numbers.

Therefore, $2x+2=10$.

Subtracting 2 from each member of the equation, gives

$$2x+2-2=10-2;$$

uniting terms, $2x=8$.

Dividing each member of the equation by 2, gives

$x=\frac{8}{2}=4$, the smaller;

then $x+2=6$, the greater.

12. The larger of two numbers is five more than the smaller, and their sum is seventeen. What are the numbers?

13. There are eight books on a shelf; a part belonging to Mary, and the remainder to Anna; but Mary owns two more than Anna. How many belong to each?

14. Divide twenty-three chestnuts between John and James, giving to James five more than to John. How many will each have?

15. "Father," says Thomas, "if I had ten cents more, I could buy a new sled; but the carpenter will not make me one for less than a dollar." How many cents has Thomas?

16. A railroad car has seats for sixty persons, and all the seats, except eight, are filled. How many passengers are in the car?

17. George and Charles together had twenty-two

cents, but George had four more than Charles. How many had each?

18. The sum of two numbers is twenty, and their difference is six. What are the numbers?

19. James travels nine miles more than William, and the distance both travel is forty-nine miles. How far does each travel?

20. The sum of two numbers is twenty-five, and the larger is seven more than the smaller. What are the numbers?

21. Put forty-one apples into two baskets, so that one basket shall contain eleven more than the other. How many will you put into each basket?

22. There are two numbers, one of which is twelve greater than the other, and their sum is thirty-two. What are the numbers?

23. Divide thirty-five into two such parts that the larger shall be nine more than the smaller. What is each part?

24. George has seven books more than Mary; and they both have twenty-nine. How many has each?

25. Two men, A and B, were thirty miles apart, and travelled towards each other till they met. They then found that A had travelled six miles more than B. What distance did each travel?

26. The difference between two numbers is thirteen, and their sum is twenty-seven. What are the numbers?

27. There are two numbers, one of which is seven more than the other, and their sum is twenty-three. What are the numbers?

28. The sum of three numbers is thirty-six. The

first is two more than the second, and the third four more than the second. What are the numbers?

Let $x =$ the second number;
then the first will be $x + 2$,
and $x + 4$ will represent the third.
Then $x + x + 2 + x + 4$ will express their sum.
But their sum is, by the question, 36;
therefore, $3x + 6 = 36$.
Subtracting 6 from each member of the equation, gives
$3x + 6 - 6 = 36 - 6$;
uniting terms, $3x = 30$.
Dividing each member of the equation by 3, gives
$x = 10$, the second number;
then $x + 2 = 12$, the first number;
and $x + 4 = 14$, the third number.

29. Three boys have thirty-two books. John has three more than James, and William two more than John. How many has each?

30. George, Charles, and Robert, together, have forty-eight pears. George has two more than Charles, and Robert has as many as George and Charles both. How many has each?

31. The sum of three numbers is fifty-two. The first is twice the second, and the second is four more than the third. What are the numbers?

32. Three men spend sixty-three dollars. A spends three dollars more than C, and C spends twice as many as B. How many dollars does each spend?

33. Divide the number forty-one into three such parts that the greatest shall be seven more than the least, and the third four more than the least. What are the parts?

SECTION V.

1. After giving away three books, George found he had seven left. How many had he at first?

Let x represent his number of books at first;

then $x-3$, that is, x less 3, will equal the number he had left.

But he had 7 books left.

Therefore, by a condition of the question, $x-3=7$.

If x, with 3 taken from it, is equal to 7, x itself must be equal to 7 and 3 more;

that is, $x=7+3$;

or $x=10$ books, the number he had at first.

Or,

adding 3 to each member of the equation,

$$x-3=7,$$

gives $x-3+3=7+3$.

Uniting terms in each member, gives

$x=10$, as above.

2. Anna lost four needles, and now has nine left. How many had she at first?

3. Charles gave seven cents to a poor woman, and then had twelve left. How many had he at first?

4. Robert lost twelve pens, and has seventeen left. How many had he at first?

5. Mary has eaten nine plums, and she still has sixteen in her basket. How many had she at first?

6. Thomas took seventeen cents from his purse, and then there were ten remaining in it. How many cents were in the purse at first?

7. What number is that, from which if eleven be taken, the remainder will be thirteen?

8. From what number must seven be taken, that thirteen only may remain?

9. Jane has three less than Mary, and both together have eleven books. How many has each?

Let x represent Mary's share;

then x less 3, that is, $x-3$, will stand for Jane's share,

and $x+x-3$, that is, $2x-3$, will represent what both have.

But both have 11 books;

therefore, $2x-3=11$.

If $2x$ less 3 is 11, $2x$ must equal 11 and 3 more, that is, $2x=11+3$, which is 14.

If $2x=14$, then x will equal $\frac{1}{2}$ of 14, which is 7;

therefore $x=7$ books, or Mary's share;

and $x-3=4$ books, or Jane's share.

Or,

since $2x-3=11$,

adding 3 to each member of the equation, gives

$$2x-3+3=11+3.$$

Uniting terms in each member, $2x=14$;

dividing each member by 2, $x=7$;

and $x-3=4$, as above.

10. Lucy and Anna had each an equal number of pictures. After three were taken from Anna, both together had only seventeen. How many had each?

Let x represent Lucy's number of pictures;

then x will also equal the number Anna had at first,

and $x-3=$ the number Anna had remaining, after three pictures were taken from her.

Then $x+x-3=$ the number both together had, after three were taken from Anna.

But both had 17, after three were taken from Anna;

therefore, by the conditions of the question,

$$2x - 3 = 17.$$

Adding 3 to each member of the equation, gives

$$2x - 3 + 3 = 17 + 3;$$

uniting terms in each member, $2x = 20$.

Dividing each member by 2, gives

$x = 10$, or Lucy's number of pictures,

and $x - 3 = 7$, or the number Anna had remaining.

11. Josephine and Georgiana had each an equal number of dolls; but Georgiana gave away five of her dolls, and now both together have but eleven. How many had each at first, and how many has each now?

12. John and William together have twenty cents. John lacks only seven of having twice as many as William. How many has each?

If $x =$ William's share, then $2x - 7 =$ John's share.

13. Eliza and Sarah bought a doll for twenty-six cents. If Sarah had paid four cents more, she would have paid twice as much as Eliza. How much did each pay?

14. A man and a boy together have thirty dollars. If the man had two dollars more, he would have three times as many as the boy. How many dollars has each?

15. The difference between two numbers is seven. Now, if x be put for the greater, what will stand for the smaller? What will represent their sum?

16. If the sum of the two numbers in the 15th question be thirteen, what will be the numbers?

17. If the sum of the two numbers in the 15th question be nine, what will be the numbers?

18. The sum of two numbers is twelve, and their difference is four. What are the numbers?

19. The sum of two numbers is thirty-three, and the greater lacks three of being equal to three times the smaller. What are the numbers?

20. The sum of two numbers is twenty-seven, and the greater lacks twelve of being twice the smaller. What are the numbers?

21. The sum of two numbers is fifty, and the greater is only ten less than four times the smaller. What are the numbers?

22. The united ages of a father and son are thirty-four years, and the father lacks only two years of being five times as old as the son. What was the age of each?

23. George and Mary together have written forty-five copies in school. If Mary had written three copies more, she would have written three times as many as George did. How many did each write?

24. The sum of two numbers is sixty-seven, and the greater lacks five of being seven times the smaller What are the numbers?

25. The united ages of a gentleman and his two sons are fifty-two years. The elder son is three times as old as the younger; the father's age lacks but eight years of being twice the sum of the united ages of both sons. What is the age of each?

26. If x less three is equal to seven, how many must be added to seven to make it equal to x?

27. If $x - 3$ be added to $x - 3$, what will be the sum?

3 less than x added to 3 less than x, will be 6 less than $2\,x$, or $2\,x$ less 6.

28. If $x-3$ be multiplied by 2, what will the product be?

29. If $x-3$ be multiplied by 3, what will the product be?

30. If $x-3$ be multiplied by 5, what will the product be?

31. If $2x-7$ be multiplied by 4, what will the product be?

32. When $x-8=12$, what must be added to 12 that the sum may be equal to x?

33. When x less 8 plus 8 is equal to 12 plus 8, what is the value of x?

SECTION VI.

1. ADELINE bought an equal number of pears and peaches for nine cents, paying one cent for each pear, and two cents for a peach. How many of each did she buy?

Let x represent the number of pears;
then x will also stand for the number of peaches.
She bought x pears, and x peaches.
She paid x times 1 cent for the pears, and x times 2 cents for the peaches.
But x times 1 cent is x cents,
and x times 2 cents, being twice as many as x times 1 cent, will equal $2x$ cents;
then $x+2x=3x$ cents, will represent the number of cents she gave for all the pears and peaches.

But she gave 9 cents for all of them.

Therefore, by the conditions of the question,

$$3x = 9.$$

Dividing each member of the equation by 3, gives

$x = 3$, the number of each that she bought.

Ans. 3 pears and 3 peaches.

2. George bought lead pencils at two cents, and slate pencils at one cent apiece, of each an equal number. They cost him fifteen cents. How many of each did he buy?

3. A boy bought as many pen-holders as writing-books; paying three cents for a pen-holder, and six cents for a writing-book. How many of each did he buy for twenty-seven cents?

4. A farmer sold as many barrels of apples as of cider, and received for the whole twenty dollars. He sold the apples at two dollars a barrel, and the cider at three dollars a barrel. How many barrels of each did he sell?

5. George and Henry together paid twenty-eight cents for oranges, and each bought an equal number But George paid at the rate of three cents, and Henry at the rate of four cents, apiece. How many oranges did each buy, and how many cents did each pay for his oranges?

6. A farmer sold as many sheep as calves; receiving two dollars for a sheep, and four dollars for a calf. He sold them all for twenty-four dollars. How many of each did he sell?

7. Two men keep their cows in a hired pasture, for which they pay forty-two dollars a year. One has two cows and the other has five cows. How much

does it cost to keep each cow, and how many dollars does each man pay?

8. A farmer gave his laborers sixty-three dollars; paying each man six dollars, and each boy three dollars. There were as many men as boys. How many boys were there?

9. Martha bought melons and oranges, of each an equal number, for forty-eight cents, giving ten cents for a melon and two for an orange. How many of each did she buy?

10. A man bought cows at eighteen dollars apiece, and sheep at two dollars, of each an equal number. How many of each did he buy for one hundred dollars?

11. Charles has his money in cents and half-dimes, of each an equal number. The whole of his money amounts to thirty-six cents. How many copper and how many silver pieces of money has he?

12. Zealous has his money, amounting to fifty-five cents, in two kinds of coin, namely, cents or copper pieces, and dimes or silver pieces, worth ten of the copper pieces. He has as many silver pieces as he has copper pieces. How many pieces of each kind has he?

13. Frederic has his money in dimes, half-dimes, and cents; and he has the same number of pieces of each kind of coin. The value of all the pieces is eighty cents. How many pieces of each kind has he?

14. Nichols bought an equal number of apples, lemons, and oranges; paying one cent for an apple, two for a lemon, and three for an orange. How many of each did he buy for twenty-four cents?

Let x represent the number of each

As he gave 1 cent for one apple, x apples cost x cents;
as a lemon cost 2 cents, x lemons cost $2\,x$ cents;
and x oranges, at 3 cents apiece, cost $3\,x$ cents.

Then $x+2\,x+3\,x=6\,x$ cents, the cost of all he bought.

But he paid for all 24 cents.

Therefore, by the conditions of the question,

$$6\,x=24;$$

and, dividing each member by 6, $x=\frac{24}{6}$;

that is, x is $\frac{1}{6}$ of 24, which is 4.

Then he bought 4 apples, 4 lemons, and 4 oranges.

15. The sum of three numbers is sixty-three. The first is twice the second, and the second is twice the third. What are the numbers?

16. A horse, saddle, and bridle, together, cost one hundred and forty-four dollars. The saddle cost twice as much as the bridle, and the horse three times as much as the saddle and bridle together. What was the cost of each?

17. A man deposited in a Savings Bank, at different times, three several sums of money, amounting to seventy dollars. The first time he put in twice as much as he did the second time, and the third time twice as much as he did the first time. How many dollars did he deposit each time?

18. A boy has peaches, pears, and apples, of each an equal number; and they cost him forty-eight cents. He gave twice as much for a pear, and three times as much for a peach, as he did for an apple; and an apple cost one cent. How many had he of each? What did all of each kind cost?

19. Three men, A, B, and C, keep their cows in the same pasture, and together pay fifty-six dollars for the use of it. A has one cow, B has three, and C has as many as A and B together. What was the cost for each cow, and what did each man pay?

20. A boy has three times as many plums as pears, and twice as many pears as peaches; in all, fifty-four How many has he of each kind?

21. Anna, Lucy, and Mary are to share sixty dollars. Anna is to have two, and Lucy three dollars, to Mary's one. How many dollars will each have?

22. Charles has some apples; George has twice as many as Charles; and Peter three times as many as George. Now, if x be put for the number which Charles has, what will stand for the respective shares of George and Peter? and what will represent the sum of their shares, or the whole number of apples?

23. If the whole number of apples in the 22d question be ninety, what will be the value of x, and how many apples has each boy?

24. If the value of x, in the 22d question, be nine, how many apples will each boy have?

25. Robert gave one half of his money for quills, at a cent apiece, and the other half for quills; at the rate of three for a cent. He bought thirty-six quills. How much money had he?

Let $x =$ one half of his money.

At a cent apiece, for x cents he bought x quills.

At 3 quills for a cent, for x cents he got three times as many quills as he did at the rate of one quill for a cent.

Then, for the other half of his money, he got $3x$ quills, and he bought, in all, x quills and $3x$ quills.

But he bought 36 quills;

therefore, $x + 3x$, or $4x = 36$.

Dividing each member by 4, $x = 9$ cents, or half of his money.

Then $2x = 18$ cents, the whole of his money.

26. A girl bought some oranges for forty-five cents, paying two cents apiece for one half of them, and three cents apiece for the other half. How many did she buy at each price, and how many in all?

27. A man bought a number of sheep for twenty-seven dollars. Half of them cost a dollar apiece, and the other half two dollars apiece. How many did he buy?

28. One man travelled four miles an hour, and another five miles. Each travelled the same number of hours. The sum of the distances which they travelled was seventy-two miles. How many hours and miles did each travel?

29. Two men are seventy miles apart, and are travelling towards each other. One travels three miles an hour, and the other four miles an hour. In how many hours will they meet? and what distance does each travel?

30. A boy bought fifty-four plums and grapes, paying an equal sum of money for each kind of fruit. The plums cost at the rate of a cent for two, and the grapes a cent for four. How many of each kind did he buy? and how much money did all cost?

SECTION VII.

1. CHARLES has half as many books as George, and they both have nine. How many books has each?

Let $x =$ George's number of books;

then Charles's number will be one half of x,

which is x divided by 2, and it may be written thus, $\frac{x}{2}$;

then $x + \frac{x}{2}$ will represent the whole number of books

But the whole number of books is 9;

therefore, by the conditions of the question,

$$x + \frac{x}{2} = 9.$$

But $x = \frac{2x}{2}$, or two halves of x,

and $\frac{2x}{2} + \frac{x}{2} = \frac{3x}{2}$;

therefore, $\frac{3x}{2} = 9$.

If three halves of x are equal to 9, one half of x must be one third of 9, which is 3, or Charles's number of books.

If one half of x is 3, the whole of x will be twice 3, which is 6, or George's number of books.

Again,

since $\frac{3x}{2} = 9$, or one half of $3x$ is 9,

the whole of $3x$ is twice 9, which is 18;

therefore, $3x = 18$.

If $3x = 18$, x will be one third of 18, which is 6

then $x = 6$, or George's number of books,

and $\frac{x}{2} = 3$, or Charles's, &c.

Or,

since $\frac{3x}{2}=9$,

multiplying each member of the equation by 2, gives

$$3x=18.$$

Dividing each member of this last equation by 3, gives

$x=6$, as above.

2. If $\frac{3x}{4}=6$, what will $\frac{x}{4}$, or one fourth of x, equal? What will x equal?

3. If each member of the equation $\frac{3x}{4}=6$ be multiplied by 4, what equation will express the product?

4. If each member of the equation $3x=24$, be divided by 3, what equation will express the quotient?

5. In the equation $\frac{2x}{5}=6$, what is the value of x? that is, what number does x represent?

6. In two classes there are fifteen pupils, and the grammar class is half as large as the reading class. How many pupils in each class?

7. Anna and Charles together have twenty-seven pens, and Charles has half as many as Anna. How many has each?

8. Robert is half as old as Mary, and the sum of their ages is twenty-one. What is the age of each?

9. In an orchard of thirty trees there are half as many cherry-trees as pear-trees. How many trees are there of each kind?

10. The sum of two numbers is eighteen, and one is half as large as the other. What are the numbers?

Let $x=$ the larger, &c.

11. Divide twenty-four into two such parts, that

one shall be half of the other. What will the parts be?

12. One number is half as large as another, and their sum is thirty-nine. What are the numbers?

13. What number must be added to half of itself, that the sum may be thirty-three?

14. What number must be added to half of itself, that the sum may be forty-two?

15. Add such a number to half of itself, that the sum may be thirty. What will the number be?

16. A number and half of the same number added together are thirty-six. What is the number?

17. What number must be added to a third part of itself, that the sum may be twenty?

18. The sum of two numbers is thirty-two, and one is a third part of the other. What are the numbers?

19. Divide thirty-five into two such parts, that one part shall be one fourth of the other.

20. A and B, in partnership, gain thirty-six dollars. A put into the firm half as much money as B, and shared proportionally in the gain. What was each one's share of the gain?

21. A man sold a knife for thirty cents, by which he gained one fourth of the cost. How much did it cost?

22. In a school of forty-five pupils, there are one fourth as many girls as boys. How many of each sex?

23. One number is one fifth of another, and their sum is forty-two. What are the numbers?

24. A man sold a horse for fifty-six dollars, by

which he gained one sixth of what the horse cost. What was the cost of the horse?

25. A put into the firm one fifth as much money as B; they gained seventy-two dollars. What was each one's share of the gain?

26. What number must be added to one eighth of itself, that the sum may be ninety?

27. The sum of two numbers is forty-eight, and one is one seventh of the other. What are the numbers?

28. If you count the lambs with the sheep, you will find one hundred and eight in the flock, and there is one eleventh as many lambs as sheep. How many of each?

29. The sum of two numbers is ninety-nine, and one is one tenth of the other. What are the numbers?

30. What number must be added to one ninth of itself, that the sum may be one hundred?

31. Mary has one sixth as many books as Anna, and they both have forty-nine. How many books has each?

32. Divide eighty into two such parts, that one part shall be one ninth of the other. What will the parts be?

33. Two men gained sixty dollars, and one gained one fifth as much as the other. What was the gain of each?

34. A man, by selling his cow for thirty-two dollars, gained one seventh of what she cost him. How many dollars did she cost?

35. The sum of two numbers is fifty-six, and one

seventh of one number is equal to the whole of the other. What are the numbers?

Let $x =$ the greater;

then $\frac{x}{7} =$ the less.

36. Two men together have twenty-five hundreds of dollars, and A has one fourth as many hundreds as B. How many hundreds has each?

Let $x =$ the number of hundreds that B has, &c.

37. A man sold a house for twenty-four hundreds of dollars, by which bargain he gained one fifth of what the house cost him. How many hundreds of dollars did he gain?

38. There were only ninety-nine sound oranges in a box bought by two boys; but Daniel paid only one eighth part as much as Frederic. How many oranges ought each to have?

39. What number must be added to one fifteenth of itself, that the sum may be sixty-four?

40. A brother and sister inherit an estate which sold for sixteen thousands of dollars; but by their father's *Will*, the brother is to have only one third as much as the sister. How many thousands of dollars will each have?

41. Divide fifty-six hundreds into two such parts, that one part shall be one seventh as many hundreds as the other. How many hundreds will each part be?

42. Anna and Mary are to share twenty chestnuts, and Mary is to have two thirds as many as Anna. How many will each have?

43. John is three fourths as old as Robert, and

the sum of their ages is twenty-eight. What is the age of each?

44. The sum of two numbers is thirty-two, and one is three fifths of the other. What are the numbers?

45. One number is five sixths of another, and their sum is seventy-seven. What are the numbers?

46. What number must be added to two ninths of itself, that the sum may be fifty-five?

47. A man sold a yoke of oxen for ninety dollars, by which he gained two sevenths of what they cost him. How much did the oxen cost? and how much did he gain?

48. In a school there are thirty-nine pupils, and there are five eighths as many studying algebra as there are studying arithmetic. How many in each study?

49. What number is that to which two thirteenths of itself must be added, that the sum may be sixty?

50. Divide fifty-four into two such parts, that one part shall be only two sevenths as large as the other. What will the parts be?

51. A horse and cow together cost ninety-six dollars, and the cow cost three fifths as much as the horse. What was the cost of each?

52. A and B, in partnership, gain eighty-four dollars. A put into the firm three fourths as much money as B, and they are to share the profits in the same proportion. How many dollars can each have?

53. The sum of two numbers is forty-five, and one number is four elevenths of the other. What are the numbers?

54. What number must be added to five ninths of itself, that the sum may be twenty-eight?

55. If one third of x be added to one third of x, how many thirds of x will the sum be? and what term will express it?

56. In $\frac{1}{4}$ of x, $\frac{2}{4}$ of x, and $\frac{3}{4}$ of x, there are how many fourths of x? and what term will express their sum?

57. What is the sum of $\frac{1}{5}$ of x, $\frac{2}{5}$ of x, and $\frac{4}{5}$ of x?

58. In x and $\frac{3}{5}$ of x there are how many fifths of x? What term will express the sum?

59. If x be added to $\frac{5}{7}$ of x, what term will express the sum?

60. What term will express the sum, if $\frac{5}{7}$ of x be added to $\frac{4}{7}$ of x?

61. If the expression, $\frac{3x}{8}+\frac{5x}{8}+\frac{7x}{8}$, be reduced to one term, what will express the sum?

62. How many ninths of x in $2x+\frac{4x}{9}+\frac{2x}{9}$? and what term will express the sum?

63. Reduce the expression $x+\frac{9x}{10}+\frac{7x}{10}$, that is, unite the terms in one term. What will that term be?

64. Reduce to one term the expression

$$x+\frac{3x}{4}+\frac{x}{4}+\frac{2x}{4}.$$

What will the term be?

SECTION VIII.

1. George has four pears, which is one half as many as Anna has How many has Anna?

Let x represent Anna's number of pears;

then $\frac{x}{2}$, or x divided by 2, equals George's pears

But George has 4 pears;

therefore, by the conditions of the question,

$$\frac{x}{2} = 4.$$

If one half of x is equal to 4, the whole of x is equal to 8;

therefore, Anna has 8 pears.

Or,

since $\frac{x}{2} = 4$,

multiplying each member of the equation by 2, gives

$x = 8$, or Anna's pears.

2. Charles has eight arrows, which is one third as many as George has. How many has George?

3. A man has ten oxen, which is two thirds of the number of his cows. How many cows has he?

4. One boy has eighteen oranges, which is three fourths as many as another boy has. How many has the other?

5. Twelve is three fifths of what number?

6. The smaller of two numbers is twenty, and it is five eighths of the larger. What is the larger number?

7. A man sold a cow for twenty-four dollars, which

was six sevenths of what she cost him. How much did she cost him, and what did he lose?

8. In a pasture there are six horses, which is two ninths of the number of sheep in the same pasture. How many sheep are in the pasture?

9. George has sixty-four cents, which is four fifths of the money in Edward's purse. How many cents are in Edward's purse?

10. One number is five eighths of another, and the smaller is thirty-five. What is the larger number?

11. There are two numbers, the smaller being seven twelfths of the other, and the smaller is twenty-one. What is the larger number?

12. A man sold his horse for seventy-two dollars, which was eight ninths of what the horse cost him. How much did the horse cost? and how much did he lose by his bargain?

13. A part of William's money is twenty-four cents, and he says it is three sevenths of all he has. How much has he?

14. The smaller of two numbers is twenty-eight, and it is seven twelfths of the larger. What is the larger?

15. Charles has thirty cents in his hand, and this is five sixths of the sum in his purse. How many cents are there in his purse?

16. One number is four, and it is two thirteenths of another. What is the other number?

17. Anna gave away six books, which was two ninths of all she had. How many had she?

18. George gave to a poor woman fourteen cents,

which was seven eighths of all he had. How many had he?

19. How many fifths of x are there in the whole of x?

20. If two fifths of x be taken from the whole of x, how many fifths of x will remain?

21. A boy, after eating two fifths of his plums, found he had twelve plums remaining. What number had he at first, and how many did he eat?

22. In the whole of x there are how many elevenths of x?

23. If from eleven elevenths of x, eight elevenths of x be taken, what part of x will remain?

24. A farmer took a load of melons to market, and, after selling eight elevenths of them, found he had twenty-one remaining. How many did he take to market?

25. Six feet of the length of a pole are under ground, and two thirds of the pole are above ground. How long is the pole?

26. Twenty-eight, the smaller of two numbers, is four ninths of the other. What is the larger number?

27. A pole is three fourths under water, and there are seven feet out of water. How long is the pole?

28. A man sold a cow for three fourths of what she cost him, and by so doing lost six dollars. What did she cost, and for how much did he sell her?

29. A man, going a journey, travelled three fifths of the distance before dinner, and by going twenty miles after dinner, he finished his journey. How long was the journey?

30. A man sold a horse for nine tenths of what he

horse cost him, and by the bargain he lost ten dollars. What did the horse cost? and for how much was he sold?

31. Eight is one third of what number?

32. Nine is three fourths of what number?

33. A boy gave away four fifths of his money, and had six cents remaining. How much money had he at first?

34. William is nine years old, and his age is three fifths of Mary's. How old is Mary?

35. A boy, by permission of his father, sold his sled for nine elevenths of what it cost him, and by doing so he lost twenty-four cents. What did the sled cost? and for how much did he sell it?

36. A man, after spending five eighths of his money, found he had twenty-one dollars remaining. How much money had he at first?

37. A girl took four ninths of her money from her purse, and then there were thirty cents remaining in it. How much money was in the purse at first?

38. A teacher, having dismissed ten of his pupils, found only three fifths of his school remaining. Of how many pupils did the school consist?

39. A man sold a carriage for four fifths of what it cost him, and by so doing he lost one hundred dollars. What did it cost him? and for how much did he sell it?

40. One number is four sevenths of another number, and the smaller is sixteen. What is the larger number?

41. Sixteen is four elevenths of what number?

42. Twenty-seven is nine tenths of what number?

43. One number is four ninths of another, and their difference is fifteen. What is the larger number?

Let $x =$ the larger number,

and $\frac{4x}{9}$ will be the smaller number.

Then $\frac{5x}{9} =$ the difference between the numbers.

But the difference is 15;

therefore, by the conditions of the question,

$$\frac{5x}{9} = 15.$$

Dividing each member by 5, gives

$$\frac{x}{9} = 3.$$

Multiplying each member by 9, gives

$$x = 27.$$

Then the larger number is 27,

and $\frac{4x}{9} = 12$, the smaller number.

44. If you saw off from a sloop's mast two ninths of its whole length, and find the remainder seventy feet long, how long was the mast at first?

45. If you take from a basket of apples two sevenths of the whole number in it, fifty apples will remain in it. How many apples were in it at first?

46. One number is three tenths of another, and their difference is forty-nine. What are the numbers?

47. Henry sold three eighths of his hens, and had fifteen remaining. How many had he before he sold any?

48. Forty-eight is six elevenths of what number?

49. Sixty-three is seven ninths of what number?

50 One man is three fourths as old as another and the difference of their ages is ten years. What is the age of each?

Let $x =$ the age of the elder, &c.

Then $\frac{x}{4} =$ the difference of their ages.

51. One drove of cattle is three fifths of another, and the difference between the two droves is twenty-four. How many are there in each drove?

52. In one flock there are seven eighths as many sheep as there are in the other flock, and the smaller flock has in it ten less than the larger. How many in each flock?

53. Charles lost three eighths of his money, and then had only forty-five cents. How much money had he before his loss?

54. One number is three fourths of another, and the difference between them is six. What are the numbers?

55. One number is four fifths of another, and if seven be added to the smaller, it will be equal to the larger. What are the numbers?

56. The difference between two numbers is twenty-four, and one is three elevenths of the other. What are the numbers?

57. To three fourths of what number must five be added that the sum may be equal to that number?

58. If to one third of some number twelve be added, the sum will be equal to the number itself. What is the number?

59. If five be added to six sevenths of a man's age, the sum will be his age. How old is he?

60. Anna's age is three fifths of Mary's, and the difference of their ages is four. What are their ages?

61. The difference between two numbers is eighteen, and one is five eighths of the other. What are the numbers?

62. Thirty is five sevenths of what number?

63. Sixty-six is eleven twelfths of what number?

64. What part of x must be added to two thirds of x, that the sum may equal the whole of x?

65. What part of x must be added to three sevenths of x, that the sum may be the whole of x?

66. In one half of x there are how many fourths of x?

67. In one third of x there are how many sixths of x?

68. How many sixths of x are there in two thirds of x?

69. How many ninths of x in one third of x?

70. Two thirds of x may be expressed thus, $\frac{2x}{3}$; that is, $2x$ divided by 3. How will you express three ninths of x?

71. How many ninths of x in two thirds of x, or $\frac{2x}{3}$?

72. In $\frac{4x}{5}$ there are how many tenths of x?

73. How many twelfths of x are there in $\frac{3x}{4}$?

74. In $\frac{2x}{7}$ there are how many fourteenths of x?

75. How many fourteenths of x are there in $\frac{x}{2}$?

76. In $\frac{2x}{3}$ there are how many eighteenths of x?

SECTION IX.

1. Anna gave away half of her books, and then had four remaining. How many books had she at first?

Let x represent the number of books she had at first,

then $\frac{x}{2}$, or one half of x, will represent what she gave away;

and $x - \frac{x}{2}$, or x less one half of x, will express the number of books she had remaining.

But she had 4 books left;

therefore, by the conditions of the question,

x less one half of $x = 4$.

Since x is equal to two halves of x, the equation is,

two halves of x less one half of $x = 4$;

then one half of $x = 4$;

consequently, the whole of x must be twice 4;

and $x = 8$, the books Anna had.

Or,

since $x - \frac{x}{2} = 4$,

multiplying each member of the equation by 2, gives

$2x - x = 8$;

uniting the terms in the last,

$x = 8$, as above

2. If from some number one half of itself be subtracted, the remainder will be eight. What is the number?

3 The difference between the whole of a number

and one half of the same number is six. What is the number?

4. Charles lost one third of his money, and had six cents remaining. How many cents had he at first?

5. If from some number one third of itself be subtracted, the remainder will be ten. What is the number?

6. After Charles had spent one fourth of his money for a lead pencil, he had twelve cents remaining in his purse. How much money did the purse contain before his purchase?

7. If from some number one fourth of itself be subtracted, the remainder will be fifteen. What is the number?

8. A man sold one sixth of his cows, and then had ten cows remaining. How many had he at first?

9. A boy eat two thirds of his oranges, and had four remaining. How many had he at first?

Let $x =$ his number of oranges;

then x *less* two thirds of x, will express the number remaining.

But he had 4 oranges remaining;

therefore, by the conditions of the question,

$$x - \frac{2x}{3} = 4.$$

Multiplying each member of this equation by 3, gives

$$3x - 2x = 12;$$

uniting terms, $x = 12$, his number of oranges.

10. If two fifths of some number be subtracted from the whole of the same number, the remainder will be nine. What is the number?

11. The difference between the whole of a number

and three fifths of the same number is eight. What is the number?

12. A man sold a cow for fifteen dollars, and by so doing he lost two fifths of what the cow cost. How much did the cow cost? and what did he lose?

13. What number is that, from which if three sevenths of itself be subtracted, the remainder will be twelve?

14. A boy spent five ninths of his money, and then had sixteen cents remaining. How much money had he at first?

15. If seven tenths of some number be subtracted from the number itself, the remainder will be fifteen. What is the number?

16. A man lost two ninths of the cost of his horse, by selling him for fifty-six dollars. What did the horse cost him? and how much did he lose?

17. The difference between the whole and five eighths of a number is eighteen. What is the number?

18. A farmer sold three eighths of his flock of sheep, and had forty sheep remaining. How many sheep were in the flock at first?

19. If from a number three fourths of itself be subtracted, the remainder will be seven. What is the number?

20. If from Caroline's age you take three fourths of her age, the remainder will be six years. How old is Caroline?

21. From what number must you take three fifths of itself, that the remainder may be twenty?

22. The difference between Samuel's age and two fifths of his age is fifteen. What is his age?

23. The difference between a number and five ninths of itself is sixteen. What is the number?

24. A man paid away three eighths of his money, and had thirty dollars remaining. How much money had he at first?

25. The difference between the whole of a number and seven tenths of it is eighteen. What is the number?

26. A boy spent four ninths of his money, and had thirty cents remaining. How much had he at first?

27. After paying a debt with three tenths of his money, a man found he had forty-two dollars remaining. How much money had he at first? and what was the debt?

28. From what number must four sevenths of itself be taken, that the remainder may be eighteen?

29. The difference between a number and one sixth of itself is ten. What is the number?

30. If one third of x be taken from the whole of x, how many thirds of x will remain?

$x = \frac{3x}{3}$, that is, three thirds of x, or $3x$ divided by 3

And $3x$ divided by 3, less x divided by $3 = 2x$ divided by 3;

or, uniting terms, $\frac{3x}{3} - \frac{x}{3} = \frac{2x}{3}$.

Ans. $\frac{2}{3}$ of x.

31. What term will express the difference between x and two thirds of x; that is, between x and $\frac{2x}{3}$?

32. If one fourth of x be taken from x, what will express the remainder?

33. If $\frac{3x}{4}$ be taken from x, what will be the remainder?

34. What is the difference between x and two fifths of x?

35. If $\frac{4x}{5}$ be taken from x, what will express the remainder?

36. If $\frac{7x}{5}$ be taken from $2x$, what term will express the remainder?

$2x = \frac{10x}{5}$; and $\frac{10x}{5} - \frac{7x}{5} = \frac{3x}{5}$, the remainder.

37. What will express the difference between $2x$ and $\frac{3x}{7}$?

38. What will represent the difference between $2x$ and $\frac{9x}{7}$?

39. If $\frac{7x}{4}$ be taken from $3x$, what will represent the remainder?

40. In the equation $-\frac{8x}{5} = 4$, what does x represent?

41. In the equation $3x - \frac{7x}{3} = 6$, what is the value of x; that is, what number does x represent?

42. In the equation $4x - \frac{5x}{2} = 9$, what is the value of x?

43. In the equation $2x - \frac{4x}{3} = \frac{3}{3}$ or $2\frac{3}{3}$, what is the value of x?

SECTION X.

1. GEORGE gave one half of his books to his sister, and one fourth of them to his brother, and then found he had given away six books. How many had George at first? and how many did he give to each?

Let $x =$ the number of books George had at first;

then $\frac{x}{2}$, or one half of x, expresses the number of books which he gave to his sister,

and $\frac{x}{4}$, or one fourth of x, expresses the number of books which he gave to his brother.

But he gave them 6 books;

therefore, by the conditions of the question,

one half of x *and one fourth of* $x = 6$;

But one half of x is equal to two fourths of x,

and two fourths of x added to one fourth of $x =$ three fourths of x;

therefore, three fourths of $x = 6$.

Since *three times* one fourth of x is 6, *once* one fourth of x is one third of six, which is two.

If one fourth of x is two, the whole of x is *four times* two, which is eight;

then $x = 8$, the books George had.

He gave one half of x books, which is four, to his sister.

He gave one fourth of x books, which is two, to his brother.

Or,

by the conditions of the question,

$$\frac{x}{2} + \frac{x}{4} = 6.$$

Multiplying each term in each member of the equation by 4, gives

$$\frac{4x}{2}+\frac{4x}{4}=24.$$

Reducing fractions and uniting terms, gives

$$3x=24.$$

Dividing each member of this last equation by 3, gives

$$\frac{3x}{3}=\frac{24}{3}, \text{ or } x=8, \text{ George's books;}$$

$$\frac{x}{2}=4, \text{ the number he gave to his sister;}$$

$$\frac{x}{4}=2, \text{ the number he gave to his brother.}$$

2. If one half of a number be added to one fourth of the same number, the sum will be nine. What is the number?

3. George expended two thirds of all his money for a writing-book, and one sixth of it for a pencil. He paid ten cents for both. How much money had he at first?

4. If half of a number be added to one fourth of the same number, the sum will be twelve. What is the number?

5. If half of a number be added to the same number, the sum will be fifteen. What is the number?

6. If two thirds of a number be added to one sixth of the same number, the sum will be fifteen. What is the number?

7. Charles eat one fourth of his chestnuts, and gave away one third of them. He then found he had fourteen less than at first. How many had he?

Let x represent the number he had;
then, by the conditions of the question,

$$\frac{x}{4}+\frac{x}{3}=14.$$

But $\frac{x}{4}=\frac{3x}{12}$, and $\frac{x}{3}=\frac{4x}{12}$,

therefore, $\frac{3x}{12}+\frac{4x}{12}=14.$

Uniting terms, $\frac{7x}{12}=14.$

Multiplying each member of the equation by 12, gives

$$7x=168.$$

Dividing each member of the last equation by 7, gives

$x=24$, the number Charles had.

Or,

since $\frac{x}{4}+\frac{x}{3}=14$;

multiplying each term of each member by 12, gives

$$\frac{12x}{4}+\frac{12x}{3}=168.$$

Reducing fractions, gives

$$3x+4x=168.$$

Uniting terms, gives

$$7x=168.$$

Dividing each member by 7, gives

$x=24$, as above.

8. If one fourth of a number be added to one third of the same number, the sum will be twenty-one. What is the number?

9. The sum of one fourth and seven twelfths of the same number is twenty. What is the number?

10. If two thirds of a number be added to half of

the same number, the sum will be fourteen. What is the number?

11. A boy expended one fourth of his money for pens, and three eighths of it for pencils. He spent fifteen cents. How many cents had he at first? and how many had he left?

12. If two fifths of some number be added to three tenths of the same number, the sum will be forty-two. What is the number?

13. Mary gave one third of her books to Jane, and one seventh of them to Lucy. She gave away twenty. How many had she at first?

14. If one third, one fourth, and one sixth of a number be added together, the sum will be fifty-four. What is the number?

15. Robert gave one fourth of his money for pencils, two fifths of it for writing-books, and one tenth of it for pens. He paid out thirty cents. How much money had he at first? How much did he pay for each?

16. If you add together $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ of some number, the sum will be forty-six. What is the number?

17. If you add together $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{9}{10}$ of some number, the sum will be sixty. What is the number?

18. The sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$ of a school is twenty-two. How many pupils are there in the school?

19. What is the sum of $\frac{x}{2}+\frac{x}{4}+\frac{3x}{8}$?

20. What will express in one term the sum of the three terms $\frac{x}{3}+\frac{5x}{6}+\frac{7x}{12}$?

21. Reduce the following expression to one term, $\frac{x}{5}+\frac{x}{2}+\frac{9x}{10}$. What will represent the sum?

22. Reduce the expression $\frac{5x}{6}+\frac{2x}{3}+\frac{x}{2}$, to one term. What will be the sum?

23. If each member of the equation $\frac{3x}{4}+\frac{x}{3}=13$ be multiplied by 3, what equation will express the product?

24. If each member of the equation $\frac{9x}{4}+x=39$, be multiplied by 4, what equation will express the product?

25. To what equation will $9x+4x=156$ be equal, when the two terms of the first member are united?

26. If each member of the equation $13x=156$, be divided by 13, what number will express the value of x?

27. Reduce the equation $\frac{x}{2}+\frac{2x}{3}=7$. What number will express the value of x?

28. Reduce the equation $\frac{x}{2}+\frac{4x}{5}=13$. What number does x represent?

29. Reduce the equation $\frac{x}{2}+\frac{2x}{3}+\frac{x}{4}=17$ What will be the number represented by x?

30. In the equation $\frac{3x}{4}+\frac{x}{6}=11$, what is the value of x?

SECTION XI

1. George, Anna, and Charles have sixteen books. Anna has two more than Charles, and George has as many as both. How many has each?

Let $x =$ the number Charles has;

then $x + 2 =$ the number Anna has, because she has 2 more than Charles,

and x added to $x + 2 =$ the number George has, because he has as many as both.

Then $2x + 2 =$ George's number;

and x *added to* $x + 2$ *added to* $2x + 2$, will express what they all have.

But they all have 16 books;

therefore, by the conditions of the question

$$x + x + 2 + 2x + 2 = 16.$$

Uniting terms, gives

$$4x + 4 = 16.$$

Subtracting 4 from each member of the equation, gives

$$4x = 12.$$

Dividing each member of the equation by 4, gives

$x = 3$, the number Charles has;

then $x + 2 = 5$, the number Anna has;

and $2x + 2 = 8$, the number of books George has.

2. Three men have twenty-two dollars. The first has three dollars more than the second, and the third has as many as both of the others. How many dollars has each?

3. The sum of three numbers is forty-four. The first is four more than the second, and the third is as large as both the others. What are the numbers?

4. Mary, Lucy, and Jane have fifty-two quills. Jane has five more than Lucy, and Mary has two more than both. How many quills has each?

5. John is six years older than James, and William's age is three years more than the united ages of both. The sum of their ages is fifty-one. What is the age of each?

6. One number is eight more than a smaller number, and a third number is four more than both, and the sum of the three numbers is sixty-eight. What are the numbers?

7. George is nine years older than William, and Herman is two years older than both. The sum of their ages is forty. What is the age of each?

8. Three men received seventy-seven dollars. A received three dollars more than B, and B received four dollars more than C. How many dollars did each receive?

9. The sum of three numbers is forty. The first is ten more than the second, and the third is seven less than the first. What are the numbers?

Let $x =$ the second number;

then $x + 10 =$ the first number.

The sum of the two is $2x + 10$.

The third number, being 7 less than the first, will be expressed by $x + 10 - 7$, or $x + 3$

But the sum of the three is 40;

therefore, $x + x + 10 + x + 3 = 40$.

Uniting the terms of the first member, gives

$$3x + 13 = 40.$$

Taking 13 from each member, gives

$$3x = 27.$$

Dividing each member by 3, gives

$x = 9$, the second number;

$x + 10 = 19$, the first number;

$x + 3 = 12$, the third number.

10. Three men added their ages, and found the sum was one hundred years. The first was twelve years older than the second, and the age of the third was eight years less than the sum of the ages of the first and second. How old was each?

11. One number is seven more than a second number, and a third is nine less than the sum of the other two. Their sum is fifty-three. What are the numbers?

12. There are ninety sheep in a flock, owned by three men. A owns twelve more than B, and C owns fourteen less than A and B both. How many sheep does each own?

13. The sum of three numbers is seventy. The first is eleven more than the third, and the second is as much as the first and third together, lacking twelve What are the numbers?

14. Three men together have eighty-two dollars. A has fifteen more than B, and C has as many as A and B both. How many dollars has each?

15. Henry, William, and Robert, together, have forty-two cents. Henry has eight more than William, and Robert has only six less than both. How many cents has each?

16. In the equation $x + 2x + 6 - 8 = 16$, what is the value of x?

17. What is the value of x in the equation $x + 3 + 2x + 8 + x = 47$?

18. What is the value of x in the equation $x+2x-13+x+4=11$?

19. Fifteen books are to be divided into three such parts, that John shall have four more, and William four less, than Robert. How many will each have?

20. Divide thirty-nine into three such parts, that the first shall be seven greater, and the second seven less, than the third. What are the parts?

21. Lucy, Mary, and Anna, received each the same number of oranges, in all eighteen. But Anna gave three of her oranges to Lucy. Then how many had each?

22. Divide twenty-three into three such numbers, that the first shall be five more, and the third three less, than the second. What are the numbers?

23. Three men are to receive thirty-eight dollars. The first is to have seven more, and the second five less, than the third. How many dollars will each receive?

24. The sum of three numbers is twenty-five. The first is four more, and the second six less, than the third. What are the numbers?

25. A, B, and C, gained twenty-seven dollars, and they are to share in the following proportions; A is to have four less than B, and C five less than B. How many dollars will each receive?

26. The sum of three numbers is twenty. The first number is three less than the second, and the second is two less than the third. What are the numbers?

27. Three men together had forty-one dollars.

The first had two less than twice the second, and th third had three more than the second. How many dollars had each?

28. Divide thirty-eight into three such parts, that the first shall be six less than twice the second, and the third four less than the second. What will the parts be?

29. Anna is three years younger than Eliza, and Eliza is seven years older than Lucy. The sum of their ages is seventeen. How old is each?

30. Divide twenty-four cents between three boys, giving the first two cents less than the second, and the second two less than the third. How many will each have?

31. Twenty-six pupils in a school are in three classes. In the first class there are four less than in the second, and in the second three less than in the third. How many pupils in each class?

SECTION XII.

1. Robert and John together have eleven cents, but Robert has two more than half as many as John How many has each?

Let x represent John's money;

then $\frac{x}{2}+2$ will represent Robert's;

and $x+\frac{x}{2}+2$ will represent the money both had.

But both had 11 cents;
therefore, by the conditions of the question,

$$x + \frac{x}{2} + 2 = 11.$$

Subtracting 2 from each member of the equation, gives

$$x + \frac{x}{2} = 9.$$

Multiplying each member by 2, gives

$$2x + x = 18.$$

Uniting terms in the first member, gives

$$3x = 18.$$

Dividing each member by 3, gives

$x = 6$ cents, or John's money;

then $\frac{x}{2} + 2 = 5$, Robert's money.

2. Three boys together have twenty-five cents. William has one fourth as many as Charles, and George has half as many as Charles and four more. How many cents has each?

3. Sarah is two years older than Eliza, and if two thirds of Eliza's age be added to Sarah's age, the sum will be twenty-two. What is the age of each?

4. What is that number, to which if you add three fourths of itself and five more, the sum will be forty?

5. If to a boy's money you add three fifths of his money and six cents more, the sum will be fifty-four cents. How much money has he?

6. If three sevenths of a number and nine more be added to the number itself, the sum will be forty-nine. What is the number?

7. Three boys have sixty quills. The second has five more than the third, and the first has three fourths as many as the third. How many has each?

8. If you add two fifths of a number and eight more to the number itself, the sum will be fifty. What is the number?

9. A boy is eight years older than his sister. If twice the brother's age be added to one seventh of the sister's, the sum will be thirty-one. How old is each?

10. The difference between two numbers is three. If half the less be added to three times the greater, the sum will be forty-four. What are the numbers?

11. A farmer has five more cows than horses. If one third of the number of horses and two more be added to the number of cows, the sum will be twenty seven. How many of each has he?

12. One number is fifteen more than another, and the sum of twice the greater, added to three fourths of the less, is sixty-three. What are the numbers?

13. William is nine years older than Robert, and the sum of twice William's age and two fifths of Robert's, is fifty-four. What is the age of each?

14. What number must be added to three sevenths of itself and five more, that the sum may be forty-five?

15. A man is seventeen years older than his sister. If his age be added to three eighths of his sister's, the sum will be sixty-one. What is the age of each?

16. The difference between two numbers is ten. If the greater be added to three fourths of the less and five more, the sum will be fifty. What are the numbers?

17. In the equation $x + 10 + \frac{3x}{4} + 7 = 31$, if the terms in the first member be united, and seventeen

subtracted from each member, what will express the equation?

18. If each member of the equation, $x+\frac{3x}{4}=14$, be multiplied by 4, what equation will express the product?

19. If each member of the equation $7x=56$, be divided by seven, what number will express the value of x?

20. In the equation $x+7+\frac{2x}{5}+3=24$, what number will express the value of x?

21. Reduce the equation $2x+\frac{2x}{3}+9=25$. What will be the value of x?

22. Reduce the equation $x+3+\frac{x}{2}+\frac{x}{4}+5=22$. What number does x represent?

SECTION XIII.

1. George has only four more books than Anna has, and yet he has twice as many as Anna. How many has each?

Let $x=$ Anna's books;
then George will have $x+4$ books.
But George has also twice as many as Anna,
therefore, George has $2x$ books.
Since $2x$ represents the books George has, and $x+4$ also represents his books, the two expressions must be equal to each other.

Therefore, by the conditions of the question,

$$2x = x + 4.$$

As x and 4 are equal to $2x$, 4 alone must be equa. to x;

therefore, Anna has 4 books,

and George has $x + 4 = 8$ books;

or, since $2x = x + 4$,

subtracting x from each member of the equation, gives

$$2x - x = x - x + 4.$$

Uniting terms, gives

$$x = 4, \text{ as above.}$$

2. Robert is twice as old as Charles, and the difference between their ages is five. What is the age of each?

3. Anna had as many dolls as Lucy; but three more were given to Lucy, and she now has twice as many as Anna. How many dolls has each?

Let $x =$ Anna's number of dolls.

Lucy had the same; then she also had x dolls,

and $x + 3 =$ Lucy's present number of dolls.

But this is twice Anna's, or $2x$;

therefore, $2x = x + 3$.

Subtracting x from each member of the equation, gives

$x = 3$, Anna's number,

and $x + 3 = 6$, Lucy's number.

4. To what number must eight be added, that the sum may be three times the number?

5. Caroline is ten years older than Mary, and Caroline's age is three times Mary's. How old is each?

6. What number must be increased by twelve, that the sum may be three times itself?

7. Lucy is fifteen years younger than Jane, and Jane is four times as old as Lucy. What is the age of each?

8. What number will be four times as large, if you add eighteen to it?

9. John and James are of the same age. If to the sum of their ages twenty-seven be added, the sum will be five times the age of either. What is the age of each?

10. What number is five less than twice itself?

11. What number is that which is eighteen less than three times itself?

12. John has thirty-three cents more than Peter, and he has four times as much money as Peter. How many cents has each?

13. What number must be added to itself, and then be increased by five, that the sum may be three times the number?

14. A boy is four times as old as his playmate; and the difference in their ages is twelve years. What is the age of each?

15. The difference between two numbers is three, and twice the larger is equal to three times the smaller. What are the numbers?

16. A father is five times as old as his son, and the difference of their ages is thirty-two. What is the age of each?

17. One number is six times another, and the difference between them is forty. What are the numbers?

18. If you give Sarah sixteen books more than you give Caroline, she will have three times as many as Caroline. How many books will each have?

19. If forty years be added to a person's age, the sum will be three times his age. What is his age?

20. The difference between two numbers is four, and three times the larger is equal to five times the smaller. What are the numbers?

21. If Charles lives seven years longer, his age will be double what it now is. What is his age?

22. If twelve be added to some number, the sum will be four times that number. What is the number?

23. If John lives fourteen years more, he will be three times as old as he is now. How old is he?

24. If twenty cents be added to a boy's money, the sum will be three times as much as it now is. How much money has he?

25. Fifteen years hence, Mary's age will be four times what it now is. What is her age?

26. If twenty be added to some number, the sum will be five times that number. What is the number?

27. If you give twenty-four cents more to James, he will have five times as many as he now has. How many has he?

28. What number must be added to itself and to seven more, that the sum may be three times that number?

29. When George shall be thirty years older, his age will be four times as much as it is now. What is his age?

30. What number must be added to itself and to twenty-seven more, that the sum may be five times that number?

31. John is four years older than Henry, and twice

John's age is three times Henry's age. What is the age of each?

32. The difference between two numbers is ten, and three times the greater is equal to five times the less number. What are the numbers?

33. One man is twelve years older than his brother, and three times the man's age is equal to five times the brother's. How old is each?

34. The difference of two numbers is six, and five times the greater is equal to eight times the less. What are the numbers?

35. A man is six years older than his wife, and seven times the man's age is equal to nine times his wife's. What is the age of each?

36. One number is nine more than another, and four times the greater is seven times the less. What are the numbers?

37. If $x+6$ be multiplied by 3, what will express the product?

38. If $2x+4$ be multiplied by 5, what will express the product?

39. In the equation $3x+12=5x$, if $3x$ be subtracted from each member, what will express the result?

40. If each member of the equation $2x=12$ be divided by 2, what equation will express the quotient?

41. In the equation $2x+15=7x$, what number will express the value of x?

42. Reduce the equation $3x+9=6x+3$. What number does x represent?

43. Reduce the equation $x+20=34$. What is the value of x?

SECTION XIV.

1. Eliza gave half of her books to Joshua, and one fourth of them to Samuel, and then had but two left for herself. How many had she at first, and how many did she give to each?

Let $x =$ the whole number of books;

then $\frac{x}{2} =$ the books she gave to Joshua,

and $\frac{x}{4} =$ the books she gave to Samuel;

then $\frac{x}{2} + \frac{x}{4} + 2$ must represent all the books she had.

Therefore, by the conditions of the question,

$$\frac{x}{2} + \frac{x}{4} + 2 = x.$$

Multiplying each member by 4, gives

$$\frac{4x}{2} + \frac{4x}{4} + 8 = 4x.$$

Reducing fractions in the first member, gives

$$2x + x + 8 = 4x;$$

uniting terms in the first member,

$$3x + 8 = 4x.$$

Subtracting $3x$ from each member, gives

$$8 = x.$$

Therefore Eliza had 8 books;

then $\frac{x}{2} = 4$, Joshua's books;

and $\frac{x}{4} = 2$, Samuel's books.

2. Sarah gave one third of her money for a book, and one sixth of it for a pencil, and then had eight cents left. How many cents had she at first?

3. If one half of a number be added to one third of the same number and four more, the sum will be the number itself. What is the number?

4. A man, being asked his age, replied, "If ten years be added to one fourth and one third of my age, the sum will be my age." How old was he?

5. If two thirds of a number and seven more be added to one sixth of the same number, the sum will be equal to the number. What is the number?

6. Daniel gave one fourth of his cherries to one boy, and one sixth of them to another, and had twenty-one left. How many had he at first?

7. What number is that which is equal to the sum of two thirds and one fourth of itself added to five?

8. A farmer kept his cows in three pastures. In one pasture he had one third of them, in another one fourth of them, and ten cows in the third pasture. How many cows had he?

9. If one fourth and one sixth of a number be added to thirty-five, the sum will be the number itself. What is the number?

10. Thomas had one fourth of his money in a purse, three eighths of it in his pocket, and nine cents in his hand. How many cents had he?

11. The sum of two fifths and three tenths of a number, is twelve less than the number itself. What is the number?

12. In an orchard, one fourth of the trees are plum-trees, one third cherry-trees, and fifteen are pear-trees. How many trees are there in the orchard?

13. What number is that which is equal to the sum of one half and one fifth of itself added to eighteen?

14. A man broke one fifth of his eggs on his way to market; while there he sold three tenths of them, and still had twenty left. How many eggs did he have when he started from home?

15. If two thirds and one ninth of a number be added to sixteen, the sum will be the number itself What is the number?

16. One sixth of the pupils of a school are in the first class, one fourth in the second, one third of them in the third, and twenty-four in the fourth class. How many pupils are there in the school?

17. The sum of one half, one fifth, and one tenth of a number, is eight less than the number itself. What is the number?

18. One third of a man's farm is used as a pasture for his cattle, one sixth is meadow, one twelfth woodland, and the remaining twenty acres he cultivates with the plough. How many acres has he?

19. If one third, one sixth, and one ninth of a number be added to fourteen, the sum will be the number itself. What is the number?

20. In a college one fifth of the students are seniors, one fourth juniors, three tenths sophomores, and there are forty students in the freshmen class. How many students are there in the college?

21. Reduce the expression $\frac{x}{2}+\frac{3x}{4}+\frac{5x}{8}$ What term will express the sum?

22. Reduce the expression $\frac{2x}{3}+\frac{3x}{4}+\frac{x}{2}$. What will express the sum?

23. If each member of the equation $\frac{2x}{3}+\frac{x}{4}+\frac{x}{6}$

$= 13$ be multiplied by 12, what will express the equation before it is reduced?

24. If the terms in the first member of the equation $\frac{24x}{3} + \frac{12x}{4} + \frac{12x}{6} = 156$, be reduced to one term, what will the equation be?

25. If each member of the equation $13x = 156$ be divided by 13, what number will show the value of x?

26. Reduce the equation $\frac{x}{2} + \frac{3x}{5} + 7 = 18$. What number does x represent?

27. Reduce the equation $\frac{x}{2} + \frac{x}{3} + \frac{x}{9} + 5 = 22$. What number will express the value of x?

SECTION XV.

1. Charles gave away one fifth of his money, and had eight cents left. How many cents had he at first?

Let x = the money Charles had;

then $\frac{x}{5}$ = the money he gave away.

But he had 8 cents left;

therefore, by the conditions of the question,

$$x - \frac{x}{5} = 8.$$

Multiplying each member of the equation by 5, gives

$$5x - x = 40;$$

uniting terms in the first member,

$$4x = 40;$$

dividing each member of the equation by 4,

$x = 10$ cents, the money Charles had.

2. A boy spent one fifth of his money, and gave away two fifths of it. He then had six cents left How many cents had he at first?

Let $x =$ the money he had at first;

then $\frac{x}{5} =$ the money he spent,

and $\frac{2x}{5} =$ what he gave away;

then $x - \frac{x}{5} - \frac{2x}{5}$ must be equal to what he had left

But he had 6 cents left;

therefore, $x - \frac{x}{5} - \frac{2x}{5} = 6.$

Since $x = \frac{5x}{5}$, then $\frac{5x}{5} - \frac{x}{5} - \frac{2x}{5} = 6.$

Uniting terms in the first member, gives

$$\frac{2x}{5} = 6.$$

If $\frac{2}{5}$ of $x = 6$, $\frac{1}{5}$ of $x = \frac{1}{2}$ of 6.

If $\frac{1}{5}$ of $x = 3$, the whole of $x = 15$ cents, the boy's money.

Or,

since $\frac{2x}{5} = 6,$

dividing each member of the equation by 2, gives

$$\frac{x}{5} = 3.$$

Multiplying each member by 5, gives

$x = 15$, as above.

Remark. — Form the equations in this section by subtracting the parts, &c.

3. Jane eat one half of her peaches, and gave

away one third of them. She had five left. How many had she at first?

4. If from some number you subtract one half of itself and one third of itself, the remainder will be seven. What is the number?

5. A boy spent one half of his money for writing-books, and one sixth of it for pencils. He had twelve cents left. How much money had he at first?

6. From what number must one third and one sixth of the same number be taken, that the remainder may be eighteen?

7. Mary lost one third of her money, and gave away two fifths of it. She had eight cents left. How many cents had she at first?

8. From what number must one half and one tenth of itself be taken, that sixteen may be left?

9. A man paid one fourth of his money for a pair of boots, and one fifth of it for a hat. He had eleven dollars left. How much money had he at first? and how much did he pay for each of his purchases?

10. A boy spent one half of his money, and gave away one fifth of it. He then had twelve cents left How many cents had he at first?

11. If from a number you subtract one fourth of itself and one third of itself, the remainder will be ten. What is the number?

12. If from a man's age one fourth and one sixth of his age be subtracted, the remainder will be four teen years. What is the man's age?

13. From what number must you take two fifths and three fifteenths of itself, that the remainder may be twenty-four?

14. A farmer sold one fourth of his flock of sheep to one man, and two fifths to another, and he still had fifty-six sheep left. How many sheep were in the flock at first?

15. If from some number you subtract one third and four ninths of itself, the remainder will be fourteen. What is the number?

16. A man sold one third of his farm to one person, and two ninths of it to another, retaining only forty-eight acres for himself. Of how many acres did the farm at first consist?

17. From what number must one half and three fourteenths of the same number be taken, that the remainder may be eight?

18. A boy gave one half of his chestnuts to one playmate, one fifth to another, and one tenth to a third, and kept twenty-four for himself. How many had he at first?

19. If one fourth, one sixth, and one twelfth of a number be subtracted from itself, the remainder will be forty-eight. What is the number?

20. A drover on his way sold one third of his cattle to one man, and one ninth to another, and then had fifty left. How many were in the drove at first?

21. If from five fifths, or the whole of x, two fifths of x be subtracted, what will remain?

22. If from x, two fifths of x and one fifth of x be subtracted, what will express the remainder?

23. If from six sixths, or the whole of x, one half of x and one third of x be subtracted, what will represent the remainder?

24. Reduce the expression $x - \frac{x}{3} - \frac{x}{4}$. What will represent it?

25. What will represent the expression $x - \frac{2x}{7} - \frac{x}{3}$, when reduced?

26. What will the expression $x - \frac{x}{5} - \frac{2x}{11}$ become when it is reduced?

27. If from $\frac{24x}{12}$, or twice the whole of x, $\frac{17}{12}$ of x be taken, what will represent the remainder?

28. What will the expression $2x - \frac{x}{2} - \frac{x}{3} - \frac{x}{4}$ become when it is reduced?

29. Reduce the expression $2x - \frac{x}{2} - \frac{3x}{4}$. What will represent it?

30. In the equation $x - \frac{x}{2} - \frac{x}{3} = 2$, if each term of the first member be changed to sixths, what will the equation be?

31. In the equation $\frac{6x}{6} - \frac{3x}{6} - \frac{2x}{6} = 2$, where the fractions are reduced to the *same denominator*, if the terms of the first member be united, what will express the equation?

32. In the equation $\frac{x}{6} = 2$, if each member be multiplied by the denominator, 6, the equation will be *cleared of fractions*. What will be the value of x?

33. If the fractions $\frac{x}{3}$ and $\frac{x}{4}$ be reduced to the *same denominator*, what will be the *common denominator*? and what will the fractions be?

34. Reduce the equation $x - \frac{x}{2} - \frac{x}{5} = 6$. What is the number represented by x?

35. If the equation $2x - \frac{x}{2} - \frac{3x}{4} = 9$ be reduced, what will be the value of x?

36. In the equation $2x - \frac{x}{2} - \frac{4x}{5} = 7$, if all the terms of the first member be reduced to a common denominator, the equation will be $\frac{20x}{10} - \frac{5x}{10} - \frac{3x}{10} = 7$. If this equation be reduced, what will be the value of x?

SECTION XVI.

1. If Charles eats one third of a barrel of apples in a week, in how many weeks will he eat three thirds or the whole barrel?

Let x represent the *time, or number of weeks,* in which he will eat the whole barrel.

If he eats $\frac{1}{3}$ of a barrel in one week, in x weeks he will eat $\frac{x}{3}$ of a barrel.

But in x weeks he eats the whole barrel;

therefore, $\frac{x}{3}$ of a barrel must be equal to the whole barrel;

that is, $\frac{x}{3} = \frac{3}{3}$ or a whole one.

If $\frac{1}{3}$ of $x = \frac{1}{3}$ of 3, the whole of x will be equal to the whole of 3:

therefore, $x = 3$,

and he will eat the barrel in 3 weeks.

Or,

by the conditions of the question, $\frac{x}{3} = 1$

Multiplying by 3, gives $x = 3$.

2. If George eats one sixth of a barrel of apples in one week, how long will six sixths, or a whole barrel, last him?

3. George eats one sixth of a barrel of apples in a week, and Charles eats one third of a barrel in the same time. In how many weeks will both together eat the whole barrel?

Let $x =$ the number of weeks in which both will eat a whole barrel.

George, in x weeks, eats $\frac{x}{6}$ of the barrel;

Charles, in x weeks, eats $\frac{x}{3}$ of the barrel;

both together, in x weeks, eat $\frac{x}{3} + \frac{x}{6}$ of the barrel;

But both, in x weeks, eat the whole barrel;

therefore, by the conditions of the question,

$$\frac{x}{3} + \frac{x}{6} = 1;$$

and reducing the fractions to the same denomination, gives

$$\frac{2x}{6} + \frac{x}{6} = 1.$$

Clearing the equation from fractions, by multiplying by 6, the denominator, gives

$$2x + x = 6;$$

uniting terms in the first member,

$$3x = 6$$

Dividing by 3, gives $x = 2$;

therefore, both together eat the barrel in 2 weeks.

4. How many times is the sum of one sixth and one third contained in one?

5. A man can dig one fourth of a trench in one week In how many weeks can he dig the whole of it?

6. A man can build one sixth of a stone wall in one day. In how many days will he build the whole wall?

7. A man can build one third of a stone wall in one day. How many days will it take him to build the whole of it?

8. One man can build one third of a wall in one day, and another man can build only one sixth of the same wall in a day. In how many days will both men, working together, build the wall?

9. If a man can eat one eighth of a barrel of bread in a week, how long will a whole barrel last him?

10. If a man can do two ninths of some stated piece of work in one day, how many days must he work to do the whole of it?

Let x represent the number of days in which he can do the whole of it.

Since he does $\frac{2}{9}$ of it in one day, in x days he will do

x times two ninths of it, which is $\frac{2x}{9}$.

But in x days he will do all of it;

therefore, $\frac{2x}{9}$ must be equal to the whole of it, or 1:

that is, $\frac{2x}{9} = 1$.

If $\frac{1}{9}$ of $2x = 1$, the whole of $2x = 9$ times 1.

If $2x = 9$, $x = \frac{9}{2}$, which is $4\frac{1}{2}$;

therefore, he would do the work in $4\frac{1}{2}$ days.

11. A man can shingle one fourth of the roof of a house in one day, and a boy can shingle one twelfth of it in a day. How many days will it take for both, working together, to shingle the roof?

12. How many times the sum of one fourth and one eighth of any thing will it take to make the whole of the same thing?

13. How many times is the sum of one fourth and one twelfth of any thing contained in the whole of it?

14. One horse eats one sixth of a ton of hay in a week, and another horse eats only one twelfth of a ton in the same time. How long will a ton of hay last both of them?

15. How many times is the sum of one sixth and one twelfth contained in one?

16. How many times is the sum of one third and one twelfth contained in two?

17. A man can do, in one week, one fourth of the work required to repair a house, and a boy can do three sixteenths of the same piece of work in the same time. In how many weeks will both, working together, do it?

18. How many times is the sum of one third and two sevenths contained in one?

19. How many times is the sum of two thirds and five twelfths contained in three?

20. Two men and a boy are employed to make a fence. One man can do one fourth of it in a day, the other can do one fifth of it in the same time, and the boy can do one twentieth of it in a day. How many days will it take them all to do it?

21. How many times is the sum of one fourth, one sixth, and one twelfth, contained in a whole one?

22. How many times is the sum of three fourths, five sixths, and seven twelfths, contained in three?

23. A father and his son have but one barrel of bread. The father eats three twentieths of a barrel in a week, and the son one tenth of a barrel in the same time. How long will it last them?

24. How many times the sum of one third, one seventh, and one twenty-first, will it take to make a whole one?

25. One man can do one third of a given piece of work in one day; another can do one eighth of the same work in a day; and a boy can do one twenty-fourth of it in the same time. How many days will it take the three, working together, to get it done?

26. Reduce $\frac{x}{3}$, $\frac{4x}{7}$, and $\frac{5x}{21}$ to the same denomination, and what will they become?

27. What will express the sum, if $\frac{7x}{21}+\frac{12x}{21}+\frac{5x}{21}$ be reduced to one term?

28. Reduce the equation $\frac{x}{4}+\frac{x}{6}+\frac{x}{12}=1$. What number does x represent?

29. If the equation $\frac{2x}{7}+\frac{x}{14}=1$, be reduced, what will express the value of x?

30. Reduce the equation $\frac{x}{3}+\frac{x}{4}+\frac{x}{6}=1$. What will be the number represented by x?

SECTION XVII.

1. If from two thirds of Catherine's age you subtract one third of her age, the difference will be four years. How old is she?

Let x represent her age;
then, by the conditions of the question

$$\frac{2}{3}x - \frac{x}{3} = 4.$$

But $\frac{2}{3}x$ less $\frac{x}{3} = \frac{x}{3}$;

therefore, $\frac{x}{3} = 4$.

If $\frac{1}{3}$ of $x = 4$, the whole of x must be 3 times 4;
then $x = 12$, or Catherine's age.

2. If from one half of a boy's money one fourth of his money be taken, three cents will remain. How many cents has he?

3. The difference between one half and one fourth of the same number is five. What is the number?

4. If from one third of uncle William's age one fifth of his age be taken, the difference will be eight years. What is his age?

5. If two fifths of some number be taken from seven tenths of the same number, the difference will be twelve. What is the number?

6. Mary's age is two thirds of her brother's age, and Jane's is four ninths of the same brother's age. The difference between Mary's age and Jane's, is eight years. How old is the brother, and each sister?

7. The difference between two thirds and four sevenths of the same number is four. What is the number?

8. If three fourteenths of some number be taken from two sevenths of the same number, the remainder will be two. What is the number?

9. A boy eat one fourth of his plums, and gave away one fifth of them. The difference between what he eat and what he gave away was three. How many had he? and how many did he give away?

10. If three eighths of some number be taken from three fourths of the same number, the remainder will be six. What is the number?

11. If from half of a man's money one seventh of his money be taken, the difference will be fifteen dollars. How many dollars has he?

12. The difference between three fourths and five sixths of the same number is nine. What is the number?

13. A man owned seven tenths of a flock of sheep. After selling two fifths of the whole flock, he had thirty sheep still belonging to him. How many sheep were in the flock before the sale?

14. If two sevenths of some number be taken from one half of the same number, the difference between the two parts of it will be twelve. What is the whole number?

15. John's money is two thirds of William's money, and Henry's is one ninth of William's. The difference between John's money and Henry's is fifteen cents. How much money has each?

16. The difference between three fourths and three

fifths of the same number is twelve. What is the number?

17. John owned two thirds of a basket of eggs, and, after selling one fifth of all there were in the basket, fourteen eggs still belonged to him. How many were in the basket at first?

18. If from two thirds of some number three sevenths of the same number be subtracted, the difference will be fifteen. What is the number?

19. Two fifths of a pole are in the water, one tenth in the mud, and the remainder out of water. There are nine feet more of it in the water than in the mud. How long is the pole?

20. The difference between two thirds and two ninths of the same number is thirty-six. What is the number?

21. Three sevenths of a flock of sheep were put in one pasture, three fourteenths in another, and the rest were sold. There were twelve more sheep in one pasture than in the other. Of how many sheep did the flock consist? how many were in each pasture? and how many were sold?

22. The difference between three fourths and seven eighths of the same number is eleven. What is the number?

23. If seven twelfths of some number be taken from the same number, the difference will be thirty. What is the number?

24. If the terms in the first member of the equation $\frac{x}{4} - \frac{x}{8} = 10$, be reduced to the same denomination, and then to one term, what will the equation be?

25. In the equation $\frac{5x}{3} = 10$, what number is represented by x?

26. Reduce the equation $\frac{2x}{3} - \frac{x}{4} = 15$ What number will express the value of x?

27. If the equation $\frac{3x}{4} - \frac{2x}{3} = 1$ be reduced, what will be the value of x?

28. Reduce the equation $\frac{x}{2} - \frac{x}{5} = 3$. What number does x represent?

29. What number will express the value of x in the equation $\frac{5x}{3} - \frac{5x}{6} = 20$?

30. What is the number represented by x in the equation $\frac{x}{2} - \frac{x}{20} = 9$?

SECTION XVIII.

1. The sum of the ages of two boys is twelve years, and the elder is twice the age of the younger What is the age of each?

Let x represent the age of the younger boy;
then $12 - x$ will express the age of the elder,
and $12 - x$ must be equal to twice x.
Then, by the conditions of the question,

$$12 - x = 2x.$$

If 12, with x taken from it, is equal to $2x$,
12 without x taken from it, must be equal to one more x;

therefore, $12 = 3x$, and $4 = x$.

Then the age of the younger boy is 4 years;

and $12 - x = 8$, the age of the elder.

Or,

since $12 - x = 2x$,

adding x to each member of the equation, gives

$$12 - x + x = 2x + x.$$

Uniting terms in each member, gives

$$12 = 3x.$$

Dividing each member by 3, gives

$4 = x$, as above.

2. The sum of two numbers is fifteen, and the greater is twice the smaller. What are the numbers?

3. If $12 - x$ be added to $12 - x$, what will express the sum?

It may be expressed thus; $12 - x + 12 - x$.

Uniting terms, $24 - 2x$, *Ans.*

4. If $12 - x$ be multiplied by 2, what will express the product?

5. If $12 - x$ be multiplied by 5, what will the product be?

6. George and Anna together have ten books, and twice George's books are equal to three times Anna's. How many has each?

Let x represent Anna's books;

then $10 - x$ will represent George's books;

twice George's books will be twice $10 - x$, which is

$$20 - 2x.$$

But twice George's are equal to three times Anna's; therefore, by the conditions of the question

$$20 - 2x = 3x.$$

Adding $2x$ to each member of the equation, gives

$$20 - 2x + 2x = 3x + 2x.$$

Uniting terms in each member, gives

$$20 = 5x.$$

Dividing each member by 5, gives

$$4 = x;$$

therefore, Anna has 4 books,

and $10 - x = 6$;

then George has 6 books.

7. The sum of two numbers is twenty, and twice the greater is three times the other. What are the numbers?

8. The sum of the ages of two boys is eighteen years, and four times the age of the younger boy will be twice the age of the other. What is the age of each?

9. Twice one number is four times another, and their sum is thirty. What are the numbers?

10. Two men are to share thirty-five dollars between them, and A is to have four times as many as B. How many dollars will each have?

Let x represent B's share;

then $35 - x$ will express A's.

11. The sum of two numbers is forty-two, and three times one number is four times the other. What are the numbers?

12. A farmer has seventy sheep in two pastures, and twice the number of sheep in the larger pasture will be equal to five times the sheep in the other. How many sheep in each pasture?

13. Three times one number is five times a smaller, and their sum is twenty-four. What are the numbers?

14 George had seventy-two cents, and lost a part

of them. Three times the number he lost is equal to six times the number he had left. How many cents did he lose?

15. The sum of two numbers is seventeen, and twice the greater is two more than three times the less. What are the numbers?

Let x represent the less number;

then $16 - x$ will represent the greater.

Twice the greater is $32 - 2x$, which is 2 more than $3x$.

Therefore, by the conditions of the question,

$$32 - 2x - 2 = 3x;$$

and, uniting terms in the first member,

$$30 - 2x = 3x.$$

Adding $2x$ to each member of the equation, gives

$$30 - 2x + 2x = 3x + 2x.$$

Uniting terms, gives

$$30 = 5x.$$

Dividing each member by 5, gives

$$6 = x;$$

therefore, 6 is the smaller number,

and $16 - x = 10$, the larger number.

16. Two men together spend one hundred dollars. Three times the money A spends is equal to seven times the money B spends. How many dollars does each spend?

17. The sum of two numbers is twenty-eight, and the greater is four less than three times the smaller. What are the numbers?

18. Divide thirty-one into two such parts, that twice the greater shall be two more than three times the less. What are the parts?

19. The sum of the ages of a man and his wife, is forty-two, and twice the man's age is six less than three times his wife's. How old is each?

20. Divide seventeen into two such parts, that twice one part shall be eight less than five times the other. What are the numbers?

21. A farm, containing twenty-six acres, belongs to two men. Three times A's part is six acres less than four times B's part. How many acres has each?

22. Divide twenty-five into two such parts, that three times one part shall be three more than five times the other. What are the parts?

23. A boy, after spending a part of his money, found he had remaining three times as much as he had spent. He had twelve cents at first. How much did he spend? and how much was left?

24. A man had thirty-two sheep. After selling a part of his flock, he found the remainder was four less than twice the number he sold. How many did he sell? and how many were left?

25. If $16 - x$ be multiplied by 2, what will express the product?

26. If $10 - x$ be multiplied by 7, what will be the product?

27. In the equation $12 - 3x + 2 = 4x$, what is the value of x?

28. Reduce the equation $26 - 2x - 6 = 3x$. What will be the value of x?

29. Reduce the equation $60 - 5x - 5 = 6x$. What does x represent?

30. In the equation $100 - 3x - 10 = 7x$ what is the value of x?

SECTION XIX.

1. If x be taken from $2x$, the remainder will be x. How many more will be left, if $x-1$ be subtracted from $2x$?

Since not the *whole* of x is to be taken from $2x$, but x *less* 1 is to be taken away, it is evident, that in taking away the whole of x, *one* more is taken away than there should be.

This *one*, then, must be put back, or *added* to the remainder.

Therefore, if $x-1$ be taken from $2x$, the remainder must be x and *one* more, or $x+1$,

because, if $x+1$ be added to $x-1$, the sum will be $2x$.

Or,

it may be expressed thus; $2x-x+1$;

uniting terms, $x+1$.

Remark. — Therefore, to express subtraction, *change* the *signs* before the quantities to be subtracted, and connect them with the quantities from which they are to be taken

2. If the expression $x-2$ be taken from $2x$, what will represent the remainder?

3. If $x-5$ be taken from $2x$, how many more will be left than there would be, if the whole of x were taken from $2x$? and what will be the remainder?

4. If $x-7$ be taken from $2x$, how much larger will the remainder be, than if the whole of x were taken from $2x$? and what will be the remainder?

5. If $2x-9$ be taken from $5x$, what will express the remainder?

6. Peter has one cent less than John. If Peter's money be subtracted from twice John's, the remainder will be seven cents. How many cents has each?

Let x represent John's money;

then $x-1$ will represent Peter's.

Therefore, if $x-1$ be taken from twice x, the difference will be equal to 7 cents.

If x be taken from $2x$, x will remain; but if *one* less be taken from $2x$, *one* more will remain.

Therefore, if $x-1$ be taken from $2x$, the difference will be $x+1$.

Then, by the conditions of the question,

$$x+1=7.$$

Subtracting 1 from each member of the equation, gives

$x=6$ cents, or John's money;

$x-1=5$ cents, or Peter's money.

Remark.— In all the examples in this section, let $x=$ the greater, &c.

7. Henry has four cents less than Robert, and if Henry's money be taken from twice Robert's, the difference will be nine cents. How much money has each?

8. The difference between two numbers is five; and if the less number be taken from twice the greater, the remainder will be seventeen. What are the numbers?

9. The price of a cow was five dollars less than the price of an ox; and if the price of the cow be

taken from twice the price of the ox, the remainder will be thirty-five dollars. What was the price of each?

10. The difference of two numbers is twenty-five; and if twice the less be taken from three times the greater, the remainder will be eighty. What are the numbers?

11. A and B gain money in trade, but A receives ten dollars less than B. If A's share be subtracted from twice B's, the remainder will be fifty-seven dollars. How much money did each receive?

12. One number is four less than another, and if twice the less be subtracted from five times the greater, the remainder will be thirty-eight. What are the numbers?

13. Two farms belong to A and B. A has twenty acres less than B. If twice A's farm be taken from three times B's number of acres, the remainder will be one hundred acres. How many acres has each?

14. One number is seven less than another, and if three times the less be taken from four times the greater, the remainder will be six times the difference between the two numbers. What are the numbers?

15. Anna is four years younger than Mary. If twice Anna's age be taken from five times Mary's, the remainder will be thirty-five years. What is the age of each?

16. One number is ten less than another. If three times the less be taken from five times the greater, the remainder will be seven times the difference of the two numbers. What are the numbers?

17. Eliza bought a doll and a book, giving three

cents less for the doll than for the book. If twice the price of her doll be taken from four times the price of her book, the remainder will be forty-six cents. What was the price of each?

18. If $3x-12$ be taken from $5x$, what will represent the remainder?

19. If $\frac{x}{2}-\frac{1}{2}$ be taken from x, what will express the remainder?

20. If $\frac{4x}{3}-\frac{3}{4}$ be taken from $2x$, what will be the remainder?

21. Reduce the equation $4x-x+9=15$. What does x represent?

22. Reduce the equation $5x-3x+18=28$. What number does x represent?

23. What number does x represent in the equation $3x-2x+4=13$?

24. What number will express the value of x in the equation $2x-x+\frac{3}{4}=2\frac{3}{4}$?

25. What is the value of x in the equation $3x-x+7=25$?

SECTION XX.

1. John and William together have twelve apples. If John's share be subtracted from twice the number that both have, the remainder will be four times William's share. How many apples has each?

Let x represent William's share;

then $12 - x$ will represent John's share.

Twice the number that both have is twice 12, which is 24.

If from 24, $12 - x$ be taken, the remainder will be four times William's share; that is, $4x$.

If the *whole of* 12 be taken from 24, x too many will be taken away, and x must be added to what is left, to give the true remainder;

thus, $24 - 12 + x$, will express the difference.

Therefore, by the conditions of the question,

$$12 + x = 4x.$$

Subtracting x from each member, gives

$$12 = 3x.$$

Dividing each member by 3, gives

$4 = x$, or William's share;

$12 - x = 8$, or John's share.

2. If twelve be taken from twenty, the remainder will be eight. What more will remain if $12 - x$ be taken from 20? and what expression will represent the remainder?

3. If $9 - x$ be taken from 15, what expression will represent the remainder?

4. If $10 - x$ be taken from 10, what will be left?

If 10 be taken from 10, nothing will remain; but if 10 *diminished* by the *number* which x *represents*, be taken from 10, it is evident that the *number* that x *represents* will be the remainder;

therefore, x will be the remainder.

It may be expressed thus; $10 - 10 + x$, which is equal to x.

5. If $10 - x$ be taken from 23, what will express the remainder?

6. If $12 - 2x$ be taken from 17, what expression will represent the remainder?

7. If, in the above question, the value of x is 4, what number will express the remainder?

8. Robert and William together have twenty cents. If twice William's money be subtracted from three times what both have, the difference will be four and a half times Robert's money. How many cents has each?

9. The sum of two numbers is thirty. If the greater be taken from twice the sum of both, the difference will be equal to four times the less number. What are the numbers?

10. The joint wages of two men for one week are eighteen dollars; and the sum that each receives is such, that if twice B's money be subtracted from three times the amount that both receive, the remainder will be two dollars less than four times the money which A receives. How many dollars does each receive?

11. Divide seven into two such parts, that the difference between the larger and twice the sum of both will be one more than three times the smaller number. What are the numbers?

12. If Anna's age be added to Susan's, the sum will be fourteen years; but, if Anna's age be taken from three times the sum of their ages, the remainder will be eight times Susan's age. How old is each?

Let $x =$ Susan's age.

13. Divide forty into two such parts, that, if the larger be taken from twice the sum of both, the smaller shall be three elevenths of the remainder. What are the two parts?

14. Andrew and Peter have eleven oranges, which they wish to divide in such a manner, that the difference between Andrew's share and twice the number of oranges shall be one orange less than four times Peter's share. How many oranges has each?

15. The sum of two numbers is sixteen. If two fifths of the greater be taken from the sum of both, the less number will be equal to one half of the difference. What are the numbers?

16. Frederic gave twenty-four cents for a book and pencil. If the price of the book be taken from twice the cost of both, the difference will be equal to four times the price of the pencil. What was the price of each?

17. Divide twenty-eight into two such parts, that, if one fourth of the greater be taken from the whole number, the difference will be twice the less number What will the parts be?

18. A cow and sheep cost thirty dollars. If the cost of the cow be taken from twice the cost of both, the remainder will be seven times the cost of the sheep. What was the cost of each?

19. Divide thirty-two into two such parts, that if four fifths of the greater be taken from twice the whole number, the remainder will be four times the less number. What are the parts?

20. A man and boy received thirteen dollars for a week's labor. If two thirds of what the man received be taken from twice the sum that both had, the difference will be five times the money which the boy received. How many dollars had each?

SECTION XXI.

1. Eliza and Anna have ten books; and one half of Anna's number of books is equal to one third of Eliza's. How many books has each?

Let x represent Eliza's number of books;
then $10 - x$ will represent Anna's number.

One half of $10 - x$ will be $5 - \frac{x}{2}$;

but this must be equal to one third of Eliza's, or $\frac{x}{3}$;
therefore, by the conditions of the question,

$$\frac{x}{3} = 5 - \frac{x}{2}.$$

Adding $\frac{x}{2}$ to each member of the equation,

$$\frac{x}{3} + \frac{x}{2} = 5.$$

Reducing the terms of the first member to the same denomination,

$$\frac{2x}{6} + \frac{3x}{6} = 5.$$

Uniting terms in the first member,

$$\frac{5x}{6} = 5.$$

If *one sixth* of $5x$ is 5, the *whole* of $5x$ will be *six times* 5, which is 30.

If $5x = 30$, x will be *one fifth* of 30, which is 6;
therefore, $x = 6$, Eliza's books,
and $10 - x = 4$, Anna's books.

Or,

since $\frac{5x}{6} = 5$.

multiplying each member by 6,

$$5x = 30.$$

Dividing each member of this last equation by 5,

$x = 6$, as above.

2. The sum of two numbers is five, and one half of the less number is equal to one third of the greater What are the numbers?

3. What is one half of the expression $10 - x$?

4. If the expression $10 - x$ be divided by 2, what will represent the quotient?

5. What is one fourth of the expression $10 - x$?

6. If the expression $10 - x$ be divided by 4, what will represent the quotient?

7. If $15 - 2x$ be divided by 3, what will the quotient be?

8. John and William have seven oranges. One half of William's number is equal to two thirds of John's. How many oranges has each?

9. The sum of two numbers is fourteen, and one half of the greater is equal to two thirds of the less number. What are the numbers?

10. A and B gained ten dollars, and one half of A's share is equal to one third of B's. How many dollars has each?

11. The sum of two numbers is eighteen, and one half of the less number is equal to one fourth of the greater. What are the numbers?

12. The sum of the ages of Sarah and Caroline is twenty-six years. One third of Sarah's age is three fourths of Caroline's. What is the age of each?

13. The sum of two numbers is thirteen, and one

third of the greater is just three fourths of the less number. What are the numbers?

14. Daniel and Levi have twenty-one dollars. If Levi's money be divided by two, the quotient will be equal to Daniel's money divided by five. How many dollars has each?

15. The sum of two numbers is sixteen, and the less number divided by three is equal to the greater divided by five. What are the numbers?

16. Divide twenty-two dollars between A and B, so that if one dollar be taken from three fourths of B's share, and three dollars be added to one half of A's money, the sums shall be equal. How many dollars will each have?

17. The sum of two numbers is thirty-three. If one sixth of the greater be subtracted from two thirds of the less number, the remainder will be seven. What are the numbers?

18. The sum of A's and B's money is thirty-six dollars. If five eighths of B's, less two dollars, be taken from three fourths of A's, the difference will be seven dollars. How many dollars has each?

19. Reduce the equation $20-\frac{2x}{5}-\frac{x}{5}+3=17$ What number will express the value of x?

20. In the equation $\frac{20}{6}-\frac{2x}{3}+8-\frac{1}{3}=5$ what number is represented by x?

SECTION XXII.

1. ANNA paid eight cents for a book and a lead-pencil, and the pencil cost two cents less than the book. What did each cost?

Let x represent the cost of the book,
and let y represent the cost of the pencil;
then $x+y$ must express the cost of both,
and $x-y$ must be the difference between the cost of the pencil and book;
therefore, $x+y=8$, and $x-y=2$.
Adding $x+y=8$ to $x-y=2$, gives
$$x+y+x-y=8+2.$$
Uniting terms in each member, gives
$$2x=10.$$
Dividing each member by 2, gives
$x=5$ cents, the cost of the book.
Putting 5, the value of x, in the place of x, in the equation $x+y=8$, gives
$$5+y=8.$$
Taking 5 from each member of this last, gives
$y=3$ cents, the cost of the pencil.

Or,

(1.) By a condition of the question, . . . $x+y=8$
(2.) By another condition, ... $x-y=2$.
(3.) Adding 2d to 1st, ... $2x=10$
(4.) Dividing 3d by 2, ... $x=5$, as above.
(5.) Substituting 5, the value of x, for x in the 1st, ... $5+y=8$.
(6.) Taking 5 from each member of 5th, ... $y=3$, as above.

2. The sum of two numbers is eleven, and their difference is three. What are the numbers?

Let x represent the greater number,
and let y represent the less number;
then $x+y$ will express their sum,
and $x-y$ will express their difference.

(1.) Then, by a condition of the question, $x+y=11$

(2.) And, by another condition, ... $x-y=3.$

(3.) Adding 2d to 1st, ... $2x=14.$

(4.) Dividing 3d by 2, . . $x=7$, the greater number.

(5.) Subtracting x from each member of 1st, ... } . . $y=11-x.$

(6.) Putting 7, the value of x, for x, in 5th, ... } . . $y=11-7.$

Then $y=4$, the smaller number.

3. The sum of the ages of two boys is twelve years, and the difference of their ages is six years. What are their ages?

4. If $x+y$ be added to $x-y$, what expression will represent their sum?

5. If $3x+y$ be added to $4x-y$, what will express the sum?

6. If $2x-3y$ be added to $3x+3y$, what will represent their sum?

7. If the equation $x-y=3$, be added to the equation $x+y=7$, what equation will express the sum of the two? and what are the respective values of x and y?

8. If the equation $x+y=10$, be added to the equation $3x-y=16$, what new equation will result from the addition? What are the respective values of x and y?

9. If $2x - y = 11$ be added to the equation $4x + y = 31$, what will be the equation expressing their sum? and what numbers do x and y respectively represent?

10. If $4x - y = 27$ be added to $3x + y = 29$, what equation will express the result? and what will be the respective values of x and y?

11. If $3x - 2y = 18$ be added to $2x + 2y = 22$, what will be the sum of the two equations? What will be the value of x, and what the value of y?

12. If $x + 3y = 28$ be added to the equation $3x - 3y = 12$, what equation will express the result? What is the value of x and y, each?

13. If the equation $y = y$ be added to the equation $x - y = 2$, that is, if y be added to each member of this last equation, what will the equation become?

14. Charles bought five peaches and two pears for seventeen cents, and found that two pears cost four cents less than two peaches. What did one of each cost?

15. There are two numbers, such that, if three times the greater be added to three times the less, the sum will be twenty-one; and if three times the less be taken from five times the greater, the remainder will be nineteen. What are the numbers?

16. If three times Anna's age be added to three times Mary's age, the sum will be thirty-three years; and three times Mary's age is thirty-seven years less than seven times Anna's. What are their respective ages?

17. Four times the sum of two numbers is twenty eight, and the difference between six times the greater

and four times the less is twelve. What are the numbers?

18. A market-man sold three melons and four peaches for thirty-eight cents, and, at the same prices, five melons would sell for forty-two cents more than he received for the four peaches. What did he receive for one of each?

19. The difference between two numbers is seven, and four times the greater added to the less is forty-three. What are the numbers?

20. A farmer bought three sheep and a cow for twenty-six dollars. At the same rate, a cow would cost four dollars less than twelve sheep. What did he pay for the cow, and what for a sheep?

21. Twice the smaller of two numbers, taken from three times the larger, leaves only fourteen; and, if the larger be added to twice the smaller, the sum will be eighteen. What are the numbers?

22. If three fourths of John's age be taken from William's age, the difference will be six years; but if five times William's age be added to three fourths of John's, the sum will be sixty-six years. What is the age of each?

23. One number is two more than twice another, and the sum of four times the larger and twice the smaller is forty-eight. What are the numbers?

24. The sum of one seventh of Anna's money and one third of George's is six cents, and if one third of George's be taken from four sevenths of Anna's, the remainder will be nine cents. How many cents had each?

25. Divide twenty into two such parts, that the dif-

ference between the smaller and three times the larger will be twenty-four. What are the parts?

26. The sum of two numbers is fourteen, and the difference between four times the greater and twice the less is twenty-six. What are the numbers?

Let $x =$ the greater number,
and $y =$ the smaller number.

(1.) By one condition of the question, . . . $x + y = 14$.
(2.) By another condition, ... $4x - 2y = 26$.
(3.) Adding 1st to itself, ... $2x + 2y = 28$.
(4.) Adding 3d and 2d, ... $6x = 54$.
(5.) Dividing each member of 4th by 6, ... $x = 9$, the greater number.
(6.) Subtracting x from each member of 1st, ... $y = 14 - x$.
(7.) Substituting 9, the value of x, in 6th, ... $y = 14 - 9$, or 5, the smaller number.

27. If $x + y = 9$ be added to $x + y = 9$, what equation will be formed?

28. If $3x - 2y = 21$ be added to $5x + 2y = 67$, what equation will express the sum? and what numbers do x and y represent?

29. If $\frac{4x}{9} - \frac{3y}{5} = 2$ be added to the equation $\frac{2x}{9} + \frac{3y}{5} = 10$, what will express the sum? and what numbers do x and y represent?

30. Add the equation $\frac{2x}{5} + \frac{y}{4} = 18$ to the equation $\frac{3x}{5} - \frac{y}{4} = 7$. What equation will express the sum? and what numbers are represented by x and y respectively?

31. If the equation $x+y=6$ be added to itself, what equation will express the sum? that is, if $x+y=6$ be multiplied by 2, what will be the product?

32. If $x+y=6$ be subtracted from $2x+2y=12$, what equation will express the remainder? that is, if the equation $2x+2y=12$ be divided by 2, what equation will express the quotient?

33. What is one third of the equation $x+y=6$?

34. If the equation $x+y=6$ be divided by 2, what equation will express the quotient?

35. If the equation $x-y=2$ be added to itself, what equation will represent the sum?

36. If the equation $x-y=2$ be multiplied by 2, what new equation will be produced?

37. If $x-y=2$ be multiplied by 4, what equation will express the product?

38. If $x-y=2$ be subtracted from the equation $2x-2y=4$, what equation will represent the remainder?

39. If the equation $2x-2y=4$ be divided by 2, what equation will represent the quotient?

40. What is one half of the equation $2x-2y=4$?

SECTION XXIII.

1. A FARMER sold to one man two sheep and three lambs for seven dollars, and to another at the same rate, one lamb and two sheep for five dollars. What did he receive for one of each?

Let $x =$ the price of a sheep,
and $y =$ the price of a lamb;
then $2x =$ the value of two sheep,
and $3y =$ the value of three lambs.

But two sheep and three lambs were sold for seven dollars;

therefore, by the first condition of the question, the first equation will be,

$$2x + 3y = 7.$$

Also one lamb and two sheep were sold for five dollars;

therefore, by the second condition of the question, the second equation will be

$$2x + y = 5.$$

Now, if $2x + y$ be taken from $2x + 3y$, $2y$ only will remain,

and $2y$ will be equal to the difference between 5 and 7;

therefore, $2y = 2$, and $y = 1$.

Since $y =$ the price of a lamb, he sold a lamb for one dollar.

In the equation

$$2x + y = 5,$$

substituting 1, the value of y, instead of y,

$$2x + 1 = 5.$$

If $2x$ and 1 more $= 5$, $2x$ alone $= 4$,

and $x = 2$.

But $x =$ the price of a sheep; therefore he sold a sheep for two dollars.

Or,

(1.) By a condition of the question, . $2x + 3y = 7$
(2.) And, by another condition, . . . $2x + y = 5$
(3.) Subtracting 2d from 1st, ... $2y = 2$

(4.) Dividing 3d by 2, ... $y = 1$.
(5.) Subtracting y from 2d, ... $2x = 5 - y$.
(6.) Substituting 1, the value of y, in the 5th, ... $2x = 5 - 1$, or 4.
(7.) Dividing 6th by 2, ... $x = 2$, as above.

2. The sum of two numbers is twelve; and if twice the greater be added to the less, the sum will be nineteen. What are the numbers?

Let $x =$ the greater number,
and $y =$ the smaller number.

(1.) By a condition of the question, . $2x + y = 19$
(2.) By another condition, ... $x + y = 12$
(3.) Subtracting 2d from 1st, ... $x = 7$, the greater number.
(4.) Subtracting x from 2d, ... $y = 12 - x$.
(5.) Substituting 7, the value of x, in 4th, ... $y = 12 - 7$, or 5, the smaller.

3. If the expression $x + y$ be taken from $3x + y$, what will represent the remainder?

4. If $2x + 2y$ be taken from $2x + 5y$, what will be the remainder?

5. If $2x + 3y$ be taken from $7x + 3y$, what will remain?

6. If $x + 4y$ be taken from $3x + 4y$, what will express the difference?

7. If $x + 2y$ be taken from $x + 9y$, what will be the difference?

8. If the equation $2x + y = 11$ be taken from the equation $2x + 9y = 35$, what equation will express the difference? and what are the values of x and y respectively?

9. If the equation $x + 2y = 13$ be subtracted from the equation $5x + 2y = 25$, what equation will express the remainder? and what are the respective values of x and y?

10. George paid sixteen cents for two pens and four pencils. Charles, buying at the same price, paid ten cents for two pens and two pencils. What was paid for a pen, and what for a pencil?

11. The sum of two numbers is thirteen; and three times the greater added to the less is twenty-seven. What are the numbers?

12. A man sold five lemons and two oranges for twenty-two cents; and again he sold, at the same rate, two oranges and three lemons for eighteen cents. What did he receive for one of each?

13. The sum of two numbers is seven. If five times the less be added to the greater, the sum will be fifteen. What are the numbers?

14. A boy paid thirty-nine cents for three lead pencils and five writing-books, and he afterwards purchased, at the same rate, three pencils and three writing-books for twenty-seven cents. What did he pay for one of each?

15. Find two such numbers, that the sum of twice the greater added to six times the less, will be thirty-six, and the sum of three times the less added to twice the greater will be twenty-four. What are the numbers?

16. Eliza bought two peaches and seven pears for twenty cents; and again, at the same rate, she bought three pears and two peaches for twelve cents. What did she pay for one of each?

17. There are two numbers such that, if five times the greater be added to three times the less, the sum will be forty-four; and if three times the less be added to the greater, the sum will be sixteen. What are the numbers?

18. Twice George's age added to five times Lucy's is forty-three years, and twice Lucy's added to twice George's is twenty-two years. How old is each?

19. Find two numbers, such that the sum of three times the greater added to eight times the less will be forty-seven, and the sum of twice the less added to three times the greater will be twenty-three. What are the numbers?

20. A man bought a cow and ten sheep for forty dollars. He then sold, at the same rate, seven sheep and a cow for thirty-four dollars. What was the price of one of each?

21. John said to Henry, "If one half of my money be added to two thirds of yours, the sum will be ten dollars." Henry replied, "If one third of my money be added to one half of yours, the sum will be seven dollars." How many dollars had each?

22. There are two numbers such, that if three fourths of the greater be added to two thirds of the less, the sum will be twenty-five; but if two thirds of the less be added to one fourth of the greater, the sum will be only fifteen. What are the numbers?

23. If the equation $3x + 3y = 21$ be subtracted from the equation $3x + 5y = 27$, what equation will represent the remainder? and what will be the respective values of x and y?

24. If $x + 2y = 16$ be taken from $9x + 2y =$

32, what equation will express the difference? and what will be the respective values of x and y?

25. If the expression $\frac{x}{2}+\frac{y}{5}$ be subtracted from the expression $\frac{x}{2}+\frac{4y}{5}$, what will represent the difference?

26. If from the equation $\frac{2x}{3}+\frac{5y}{7}=14$, the equation $\frac{2x}{3}+\frac{2y}{7}=8$ be taken, what equation will result from the subtraction? and what will be the respective values of x and y?

27. Subtract $\frac{x}{8}+\frac{3y}{4}=11$ from $\frac{7x}{8}+\frac{3y}{4}=23$. What will be the remainder? and what numbers do x and y respectively represent?

28. Reduce the equations $3x+7y=29$, and $x+2y=9$, the respective values of x and y being the same in each equation. What number does x and y each represent?

(1.) . . . . . $x+2y=9.$

(2.) . . . . . $3x+7y=29.$

(3.) Multiplying 1st by 3, ... $3x+6y=27.$

(4.) Subtracting 3d from 2d, ... $y=2.$

(5.) Taking $2y$ from each member of the 1st, ... $x=9-2y.$

(6.) Substituting 2, the value of y, in 5th, ... $x=9-4$, or $x=5.$

Therefore, the number represented by y is 2, and the number represented by x is 5.

29. If x and y respectively represent the same

numbers in the equations $5x+2y=26$, and $5x+8y=44$, what will be the value of each?

30. If $x+y=7$ be taken from $2x+4y=20$, what will express the difference?

31. If $2x+4y=20$ be divided by 2, what will be the quotient?

32. What is one half of $4x+6y=34$?

33. Divide $3x+9y=39$ by 3. What will be the quotient?

34. What is one third of $6x+3y=33$?

35. What is two thirds of $6x+3y=33$?

36. What is three fourths of $8x+12y=28$?

37. Divide $x+y=6$ by 2. What will be the quotient?

38. What is one third of $2x+y=12$?

39. What is two thirds of $2x+y=12$?

40. What is three fourths of $2x+y=12$?

SECTION XXIV

1. A boy bought five oranges and two lemons for twenty-six cents, and a lemon cost one cent less than an orange. What was the price of one of each?

Let $x=$ the price of an orange,
and $y=$ the price of a lemon.

(1.) By a condition of the question, ... $x-y=1$
(2.) By another condition, ... $5x+2y=26.$
(3.) Multiplying 1st by 2, ... $2x-2y=2.$
(4.) Adding 2d and 3d, ... $7x=28$

(5.) Dividing 4th by 7, . . $x = 4$, price of an orange.
(6.) Subtracting $5x$ from each member of 2d, ... } . . . $2y = 26 - 5x$.
(7.) Substituting 4, the value of x, for x, in 6th, . . } . $2y = 26 - 20 = 6$.
(8.) Dividing 7th by 2, . . . $y = 3$, price of a lemon.
An orange cost 4 cents, and a lemon 3 cents.

2. The difference of two numbers is three, and the sum of four times the greater added to three times the less is twenty-six. What are the numbers?

Let $x =$ the greater number,
and $y =$ the smaller number.

(1.) By a condition of the question, ... $x - y = 3$.
(2.) By another condition, ... $4x + 3y = 26$.
(3.) Multiplying 1st by 3, ... $3x - 3y = 9$.
(4.) Adding 2d to 3d, ... $7x = 35$.
(5.) Dividing 4th by 7, ... $x = 5$, the greater.
(6.) Subtracting $4x$ from 2d, ... $3y = 26 - 4x$.
(7.) Substituting 5, the value of x, for x, in 6th, ... } $3y = 26 - 20 = 6$.
(8.) Dividing 7th by 3, ... $y = 2$, the smaller.
The greater number is 5, and the smaller 2.

3. If twice the expression $x - y$ be added to the expression $3x + 2y$, what will represent the sum?

4. If five times the expression $2x - y$ be added to $3x + 5y$, what will express the sum?

5. If twice the equation $x - y = 1$ be added to the equation $3x + 2y = 8$, what equation will represent the sum? and what will be the respective values of x and y? $2 - y = 1$ $y = 1$.

6. If three times the equation $2x - y = 5$ be

added to the equation $4x + 3y = 25$, what new equation will result from such addition? and what numbers do x and y respectively represent?

7. Multiply the equation $2x - 2y = 4$ by 2, and add the product to the equation $3x + 4y = 20$. What will represent the result? and what are the respective values of x and y?

8. If four times the equation $x - y = 5$ be added to $2x + 4y = 22$, what equation will be formed? What will be the value of x, and what of y?

9. If three times the equation $2x - 2y = 4$ be added to $4x + 6y = 68$, what equation will result? and what will be the value of each of the unknown quantities x and y?

10. A farmer sold a cow and a calf for twenty-five dollars. At the same rate, three calves would be sold for five dollars less than what he obtained for the cow. How many dollars did he get for each?

11. There are two numbers, such that, if twice the greater be taken from three times the less, the remainder will be two, and if four times the less be added to the greater, the sum will be twenty-one. What are the numbers?

12. If three times Eliza's age be added to four times Clara's, the sum will be thirty-eight, and if Clara's be taken from twice Eliza's, the remainder will be seven. What is the age of each?

13. Find two numbers, such that the sum of three times the greater and twice the less shall be twenty-one, and, if eight times the less be subtracted from nine times the greater, the remainder will be twenty-one. What are the numbers?

14. If three times George's books be added to Mary's, the sum will be twenty-three; but if five times Mary's be taken from five times George's, the remainder will be twenty-five. How many books has each?

15. Divide seventeen into two such parts, that the difference between three times the smaller and five times the larger shall be forty-five. What are the parts?

16. Two men in partnership divide their gain, so that the sum of twice A's share, added to B's share, will be twenty-seven dollars; and if three times B's money be taken from four times A's, nineteen dollars will be left. How many dollars will each have?

17. There are two numbers whose difference is four, and if three times the greater be added to three times the less, the sum will be thirty. What are the numbers?

18. A melon and an orange together cost twenty cents, and one fourth of an orange cost three cents less than one fourth of a melon. What did each cost?

Let $x =$ the cost of a melon,
and $y =$ the cost of an orange.

(1.) By a condition of the question, . . . $\frac{x}{4} - \frac{y}{4} = 3.$

(2.) By another condition, ... $x + y = 20.$

(3.) Multiplying 1st by 4, ... $\frac{4x}{4} - \frac{4y}{4} = 12.$

(4.) Reducing fractions in 3d, ... $x - y = 12.$

(5.) Adding 2d and 4th, ... $2x = 32.$

(6.) Dividing 5th by 2, ... $x = 16.$

(7.) Taking x from each member in the 2d, ... $y = 20 - x.$

(8) Substituting 16, the value of x, for x, in the 7th, ... $y = 20 - 16 = 4$.

A melon cost 16 cents, and an orange 4 cents.

19. There are two numbers, such that, if one fifth of the smaller be taken from one fifth of the larger, the remainder will be one, and if three times the larger be added to the smaller, the sum will be thirty-five. What are the numbers?

20. If one half of the price of a horse be added to one fourth of the price of a cow, the sum will be forty-six dollars; but if one eighth of the price of the cow be taken from one eighth of the price of the horse, the remainder will be seven dollars. What is the price of each?

21. If one sixth of a less number be taken from one third of a larger, the remainder will be three, and if two thirds of the larger be added to one half of the smaller, the sum will be sixteen. What are the numbers?

22. A man said, that if one half the price of his saddle were taken from one fifth of the price of his horse, the difference would be fifteen dollars; but one tenth of the price of his horse and one tenth of the price of his saddle together would be eleven dollars. What was the price of each?

23. If one half of the greater number be added to the whole of the less, the sum will be seven; but if one half of the less be taken from the whole of the greater, the remainder will be four. What are the numbers?

24. If twice $2x - y = 7$ be added to $3x + 2y$

$= 21$, what equation will express the result? and what are the respective values of x and y?

25. If four times the equation $\frac{x}{4} - \frac{y}{4} = 1$ be added to $x + y = 20$, what equation will represent the sum? and what are the respective values of x and y?

26. If three times the equation $x - \frac{y}{3} = 7$ be added to $4x + y = 42$, what equation will express the sum? What do x and y represent?

27. If four times the equation $x - \frac{y}{3} = 9$ be added to the equation $5x + \frac{y}{2} = 54$, what equation will be formed? and what will be the respective values of x and y?

28. If three times the equation $\frac{2x}{3} - \frac{y}{6} = 10$ be added to twice the equation $\frac{x}{2} + \frac{y}{4} = 12$, what equation will express the sum? and what will be the respective values of x and y?

29. If four times the equation $x - \frac{y}{4} = 4$ be added to three times the equation $2x + \frac{y}{3} = 11\frac{1}{3}$, what equation will result? and what will be the respective values of x and y?

SECTION XXV.

1. A FARMER sold five barrels of pears and four barrels of apples for twenty-three dollars. He afterwards sold, at the same rate, two barrels of each for ten dollars. What was the price of a barrel of each?

Let $x =$ the price of a barrel of pears,
and $y =$ the price of a barrel of apples.

(1.) By one condition of the question, $2x + 2y = 10$
(2.) By another condition, ... $5x + 4y = 23.$
(3.) Multiplying 1st by 2, ... $4x + 4y = 20.$
(4.) Subtracting 3d from 2d, ... $x = 3.$
(5.) Taking $2x$ from each member of 1st, ... $2y = 10 - 2x.$
(6.) Dividing 5th by 2, ... $y = 5 - x$
(7.) Substituting 3, the value of x, in the 6th, ... $y = 5 - 3 = 2.$

A barrel of pears cost 3 dollars, and a barrel of apples cost 2 dollars.

2. There are two numbers, such that three times the greater added to the less is fourteen, and three times the less added to the greater is ten. What are the numbers?

3. If twice the expression $x + y$ be subtracted from the expression $2x + 3y$, what will represent the remainder?

4. If $2x + y$, multiplied by 4, be subtracted from $8x + 6y$, what will represent the remainder?

5. If twice the equation $3x + 2y = 8$ be subtracted from the equation $7x + 4y = 18$, what equa-

tion will represent the remainder? What number does x and y each represent?

6. If from three times the equation $2x+y=10$ the equation $4x+3y=22$ be taken, what equation will express the result? What will be the respective values of x and y?

7. Multiply the equation $x+2y=7$ by four, and from the product subtract the equation $4x+4y=16$. What equation will express the result? What will be the values of x and y, each?

8. When twice Mary's age is added to three times Jane's, the sum is nineteen; and when three times Mary's age is added to Jane's, the sum is eighteen. What is the age of each?

9. When the greater of two numbers is added to twice the less, the sum is fourteen; and when the less is added to twice the greater, the sum is sixteen. What are the numbers?

10. Martha bought three pencils and a book for nineteen cents. Abby bought, at the same rate, a pencil and two books for twenty-three cents. What was the cost of one of each?

11. There are two numbers, such that, if five times the greater be added to four times the less, the sum will be thirty-eight, and if twice the less be added to the greater, the sum will be ten. What are the numbers?

12. A farmer sold five sheep and four lambs for twenty-three dollars. He afterwards bought, at the same rate, a sheep and two lambs for seven dollars What was the price of one of each?

13. Divide twenty-five into two such parts, that

the sum of three times the less and four times the greater shall be only five less than four times the sum of both parts. What are the parts?

14. If John gives you two thirds of his apples, and Henry gives you one half of his, you will receive twelve; but if John gives you one sixth of his, and Henry gives you one fourth of his, you will get only four. How many apples has each?

Let $x =$ John's number of apples,
and $y =$ Henry's number of apples.

(1.) By one condition of the question, . $\frac{x}{6} + \frac{y}{4} = 4.$

(2.) By another condition, ... $\frac{2x}{3} + \frac{y}{2} = 12.$

(3.) Multiplying 1st by 2, ... $\frac{x}{3} + \frac{y}{2} = 8$

(4.) Subtracting 3d from 2d, ... $\frac{x}{3} = 4.$

(5.) Multiplying 4th by 3, ... $x = 12$

(6.) Taking $\frac{x}{3}$ from each member of 3d, $\frac{y}{2} = 8 - \frac{x}{3}$

(7.) Putting 12 for x in the 6th, . $\frac{y}{2} = 8 - \frac{12}{3} = 4.$

(8.) Multiplying 7th by 2, ... $y = 8$

John had 12, and Henry had 8 apples.

15. The sum of two thirds of the greater of two numbers added to the less is twelve; but the sum of one fourth of both is only four. What are the numbers?

16. If from twice the equation $2x + y = 17$, the equation $3x + 2y = 27$ be subtracted, what equation will express the difference? What will be the respective values of x and y?

17. If twice the equation $\frac{x}{4}+\frac{y}{2}=11$ be taken from $x+y=30$, what will be the difference? and what the values of x and y respectively?

18. If the equation $\frac{x}{5}+\frac{y}{3}=6$ be multiplied by 3, and the product subtracted from the equation $x+y=24$, what equation will represent the remainder? and what will be the respective values of x and y?

19. If the equation $\frac{x}{7}+\frac{y}{8}=3$ be multiplied by 4, and the product be taken from the equation $\frac{6x}{7}+\frac{y}{2}=16$, what will represent the remainder? What will be the value of x, and what of y?

20. Multiply the equation $\frac{x}{3}+\frac{y}{4}=5$ by 4, from the product subtract the equation $\frac{2x}{3}+y=14$, and what will express the remainder? What will be the respective values of x and y?

21. If three times the equation $\frac{2x}{7}+\frac{2y}{9}=8$ be taken from the equation $\frac{9x}{7}+\frac{2y}{3}=33$, what equation will result? What will be the value of x, and what of y?

22. If the equation $\frac{x}{3}+\frac{y}{2}=4$ be multiplied by 4, and the product be subtracted from $\frac{5x}{3}+2y=18$, what equation will result? and what will be the respective values of x and y?

SECTION XXVI.

1. THOMAS bought three apples and one peach for five cents. Again, at the same rate, he bought six apples and three peaches for twelve cents. What was the cost of one of each?

Let $x =$ the cost of an apple,
and $y =$ the cost of a peach.

(1.) By one condition of the question, $6x + 3y = 12$.
(2.) By another condition, ... $3x + y = 5$
(3.) Dividing 1st by 3, ... $2x + y = 4$
(4.) Subtracting 3d from 2d, ... $x = 1$
(5.) Taking $3x$ from each member of 2d, ... $y = 5 - 3x$
(6.) Substituting 1, the value of x, for x, in 5th, ... $y = 5 - 3$, or 2.

An apple cost 1 cent, and a peach 2 cents.

2. There are two numbers, such that, if six times he less be added to twice the greater, the sum will be thirty-eight, and if twice the less be added to the greater, the sum will be fifteen. What are the numbers?

3. If the expression $6x + 3y$ be divided by 3 what expression will represent the quotient?

4. What is one third of the expression $6x + 3y$?

5. If the expression $9x + 6y$ be divided by 3, what will be the quotient?

6. What is one third of $9x + 6y$?

7. What will be the quotient of $8x + 4y$ divided by 4?

8. What is one fourth of $8x + 4y$?

9. If the expression $4x+2y$, divided by 2, be subtracted from $3x+y$, what will be the remainder?

10. If the expression $5x+5y$, divided by 5, be taken from $3x+y$, what will be the remainder?

11. If the equation $6x+2y=28$ be divided by 2, and the quotient be subtracted from the equation $5x+y=22$, what equation will represent the remainder? What will be the respective values of x and y?

12. If one third of the equation $6x+6y=48$ be subtracted from $5x+2y=34$, what equation will represent the remainder? and what are the respective values of x and y?

13. If you divide the equation $4x+8y=20$ by 4, and subtract the quotient from $2x+2y=8$, what equation will express the result? What will be the value of each of the quantities x and y?

14. Divide $3x+3y=24$ by 3, and subtract the quotient from $4x+y=23$. What will be the remainder? What will be the value of x, and what of y?

15. A man sold five bushels of wheat and five of rye for fifteen dollars; and again, at the same rate, two of wheat and one of rye for five dollars. What was the price of a bushel of each?

16. Divide some unknown number into two such parts, that, if three times the greater be added to the less, the sum will be seventeen, and the sum of four times the greater added to twice the less, will be twenty-four. What are the parts? and what is the number?

17. A man bought a saddle and bridle, and, being

asked what he gave for each, replied, "If four times the price of the saddle be added to twice the price of the bridle, the sum will be forty-eight dollars; and if three times the price of the saddle be added to the price of the bridle, the sum will be thirty-four" What did he pay for each?

18. A farmer, being asked how many cows and sheep he had, replied, "Two fifths of my cows and two thirds of my sheep would be ten; but one third of my sheep and the whole of my cows would be thirteen." How many had he of each?

19. There are two numbers, such that, if three fourths of the greater be added to six fifths of the smaller, the sum will be twenty-four; but if two fifths of the smaller be added to one half of the greater, the sum will be only one half as much. What are the numbers?

20. If six fifths of Daniel's age be added to three halves of Levi's, the sum will be twenty-seven years; and if the whole of Levi's age be added to three fifths of Daniel's, the sum will be fifteen. What is the age of each?

21. Divide twenty-three into two such parts, that, if three sevenths of the greater be added to two thirds of the smaller, the sum will be twelve. What are the parts?

22. A man, being asked what he gave for his horse and cow, answered, "Four sevenths of the cost of the horse, added to eight ninths of the cost of the cow, will be fifty-two dollars; and two ninths of the cost of the cow, added to two sevenths of the cost of the horse, will be twenty-two dollars." What was the cost of each?

23. If you divide the expression $4x + \frac{2y}{3}$ by 2, and subtract the quotient from $3x + \frac{y}{3}$, what will represent the remainder?

24. What will remain after subtracting one half of $4x + \frac{2y}{3}$ from $5x + \frac{y}{3}$?

25. If you divide $\frac{6x}{5} + \frac{3y}{2}$ by 3, and subtract the quotient from $\frac{4x}{5} + \frac{y}{2}$, what will express the remainder?

26. If you take one third of $\frac{6x}{3} + \frac{3y}{2}$ from $\frac{2x}{3} + y$, what will express the remainder?

27. If you divide the equation $4x + \frac{6y}{5} = 20$ by 2, what equation will express the quotient?

28. What is one fourth of the equation $\frac{3x}{5} + \frac{4y}{3} = 24$?

29. If you divide the equation $\frac{3x}{3} + \frac{4y}{3} = 20$ by 4, and then multiply the quotient by 3, what equation will represent the result?

30. What is three fourths of the equation $\frac{3x}{5} + \frac{4y}{5} = 32$?

31. If you divide the equation $\frac{2x}{3} + \frac{4y}{3} = 10$ by 2, and then subtract the quotient from $\frac{x}{3} + y = 6$, what equation will represent the remainder? What will be the respective values of x and y?

32. If one third of the equation $3x + \frac{6y}{4} = 27$ be

subtracted from $x+\frac{5y}{4}=12$, what equation will express the remainder? and what will be the respective values of x and y?

33. If from $2x+\frac{3y}{4}=24$ three fourths of the equation $x+y=17$ be taken, what equation will represent the remainder? What will be the value of x? and what of y?

SECTION XXVII.

1. Five lemons and two pears were bought for seventeen cents. At the same rate, four pears would cost fourteen cents less than six lemons. What did one of each cost?

Let $x=$ the price of a lemon,
and $y=$ the price of a pear.

(1.) By one condition of the question, $6x-4y=14$
(2.) By another condition, ... $5x+2y=17.$
(3.) Dividing 1st by 2, ... $3x-2y=7.$
(4.) Adding 3d to the 2d, ... $8x=24.$
(5.) Dividing 4th by 8, ... $x=3.$
(6.) Taking $5x$ from 2d, ... $2y=17-5x.$
(7.) Substituting 3, the value of x, in the 6th, ... $2y=17-15$, or 2.
(8.) Dividing 7th by 2, ... $y=1.$

A lemon cost three cents, and a pear one cent.

2. There are two numbers, such that, if three times the greater be added to nine times the less,

the sum will be thirty-six; and if three times the less be taken from four times the greater, the remainder will be eighteen. What are the numbers?

3. Four times John's money is six dollars more than twice Mary's, and the two together have nine dollars. How many dollars has each?

4. If the expression $4x+4y$ be divided by 4, and the quotient added to $2x-y$, what will express the sum?

5. If the expression $8x+2y$ be divided by 2, and the quotient be added to the quotient of $6x-3y$ divided by 3, what will represent the result?

6. If the equation $6x+3y=24$ be divided by 3, and the quotient be added to $3x-y=7$, what equation will express the result? and what will be the respective values of x and y?

7. If the equation $6x-4y=16$ be divided by 2, and the quotient be added to $3x+2y=16$, what equation will result? and what will be the values of x and y, respectively?

8. The difference between six times Sarah's age and three times Eliza's is eighteen years, and the sum of their ages is one half of the above difference. What is the age of each?

9. Find two such numbers, that if four times the less be taken from eight times the greater, the remainder will be twelve, and the sum of three times the greater added to the less will be seven. What are the numbers?

10. A man sold nine sheep and six calves for forty-eight dollars, and he received four dollars less for

three sheep than for two calves. What did he obtain for one of each?

11. There are two numbers, such that the sum of five times the greater added to ten thirds of the less is fifty-five, and if two thirds of the less be subtracted from the whole of the greater, the remainder will be three. What are the numbers?

12. A man bought a saddle and bridle, and said that three times the cost of the saddle, added to three fourths of the cost of the bridle, would be fifty-seven dollars, and that, if one fourth of the cost of the bridle be taken from twice the cost of the saddle, the difference would be thirty-two dollars. What did he give for each?

13. Find two such numbers, that when four times the less is added to four fifths of the greater, the sum will be twenty, and when one fifth of the greater is taken from the less, the remainder shall be one. What are the numbers?

14. A man said, that the sum of four fifths of the value of his horse, added to two thirds of the value of his cart, was forty-two dollars, and that the difference between one third of the value of his cart and three fifths of the value of his horse, was nineteen dollars. What was the value of each?

15. Divide some number into two such parts, that, if six sevenths of the greater be added to three fifths of the less, the sum shall be eighteen, and if one fifth of the less be taken from five sevenths of the greater, the remainder shall be eight. What is the number? and what are the parts?

16. A chaise and harness were sold at such prices, that if from three fourths of the price of the chaise three fifths of the price of the harness be taken, the difference will be sixty dollars, and one fifth of the price of the harness, added to one half of the price of the chaise, will be fifty-five dollars. For how much was each sold?

17. If the expression $9x+6y$ be divided by 3, and the quotient added to $x-2y$, what will be the sum?

18. What is one third of the expression $9x+6y$?

19. What is two thirds of the same expression?

20. What is one tenth of the same expression?

21. What is three tenths of the same expression?

22. If the equation $\frac{14x}{5}+\frac{7y}{2}=42$ be divided by 7, what equation will represent the result?

23. What is one fourth of the equation $\frac{8x}{3}+\frac{12y}{5}=40$?

24. What will represent three fourths of the same equation?

25. If the equation $\frac{4x}{3}-\frac{2y}{5}=18$ be divided by 2, and the quotient added to the equation $\frac{4x}{3}+\frac{y}{5}=27$, what will represent the sum? and what will be the respective values of x and y?

26. If three fourths of the equation $x-y=4$ be added to $\frac{5x}{4}+\frac{3y}{4}=21$, what equation will represent the sum? What will be the value of x, and what of y?

SECTION XXVIII.

1. A MAN weighed two kinds of cannon balls of different weights. When he put into the scale one of the heavier and two of the lighter, it took seven one-pound weights to balance them; but two of the heavier and one of the lighter weighed eight pounds What was the weight of one of each kind?

Let $x =$ the heavier ball,
and $y =$ the lighter ball.

(1.) By one condition of the question, . $x + 2y = 7$.

(2.) By another condition, ... $2x + y = 8$.

(3.) Taking $2y$ from each member of 1st, ... $x = 7 - 2y$.

(4.) Taking y from each member of 2d, $2x = 8 - y$.

(5.) Dividing 4th by 2, ... $x = 4 - \frac{y}{2}$.

Since each of the expressions $7 - 2y$ and $4 - \frac{y}{2}$ is equal to x, they are equal to each other, and they form a new equation, in which y is the only unknown quantity.

(6.) Putting $4 - \frac{y}{2}$, the value of x, for x, in 3d, . . $4 - \frac{y}{2} = 7 - 2y$.

(7.) Multiplying each member of 6th by 2, . . $8 - y = 14 - 4y$.

(8.) Adding $4y - 8$ to each member of 7th, . . . $8 - y + 4y - 8 = 14 - 4y + 4y - 8$.

(9.) Reducing 8th by uniting terms, ... $3y = 6$.

(10.) Dividing each member of 9th by 3, . . $y = 2$.

(11.) Putting 2, the value of y, for y, in the 3d, . . . $x = 7 - 4$, or 3.

Therefore, one ball weighed 3 lbs.;
the other ball weighed 2 lbs.

2. There are two numbers, such that the sum of twice the greater added to three times the less is eleven, and three times the greater is ten more than twice the less. What are the numbers?

Let x = the greater number,
and y = the less number.

(1.) By one condition of the question, $2x + 3y = 11$.

(2.) By another condition, ... $3x = 2y + 10$

(3.) Subtracting $3y$ from each member of 1st, ... $2x = 11 - 3y$.

(4.) Dividing each member of 3d by 2, ... $x = \frac{11 - 3y}{2}$

(5.) Dividing each member of 2d by 3, ... $x = \frac{2y + 10}{3}$

(6.) Putting value of x in 5th equal to value of x in 4th, $\frac{2y + 10}{3} = \frac{11 - 3y}{2}$

(7.) Multiplying each member of 6th by 6, and reducing, $4y + 20 = 33 - 9y$.

(8.) Adding $9y$, and subtracting 20 in 7th, $13y = 13$.

(9.) Dividing by 13 in 8th, ... $y = 1$.

(10.) Putting 1, the value of y, for y, in 4th, ... $x = \frac{11 - 3}{2}$, or 4.

Therefore, the greater number is 4,
and the less number is 1.

3. If $5y$ be subtracted from each side or member of the equation $3x + 5y = 19$, what equation will represent the result?

4. If the equation $3x = 19 - 5y$ be divided by 3, what equation will express the quotient?

5. What must be done to the equation $3x + 5y = 19$, that the value of x may be found in terms of the equation? and what expression will be equivalent to x, or represent the value of x?

6. If $x = \frac{19 - 5y}{3}$, what will represent twice x? What six times x?

7. In the equation $3x + 5y = 19$, what expression will represent the value of y? What of $3y$?

8. In the equation $5x - y = 13$, if y be added to each member, what equation will express the result?

9. If the equation $5x = 13 + y$ be divided by 5, what equation will represent the quotient?

10. What must be done to the equation $5x - y = 13$, that the value of x may be found in terms of the equation? What expression will be equivalent to x?

11. In the equation $4x + 3y = 22$, what expression will represent the value of x? what the value of $2x$?

12. In the same equation, what will express the value of y? what the value of $6y$?

13. Find expressions for the value of x in the equation $3x + 2y = 16$, and in the equation $2x + 5y = 18$. What are the expressions?

14. In the new equation formed by these equivalent expressions, what will be the value of y?

15. If you substitute, in the equation $2x + 5y = 18$, the value of y, thus found, in the place of y, what will be the value of x?

16. From the equations $4x + 3y = 26$, and $3x - y = 13$, make an equation which shall not contain the quantity x. What will represent that equation?

17. In the new equation thus formed, what will be the value of y?

18. If the numerical value of y, thus found, be substituted for y in the equation $4x + 3y = 26$, what will be the value of x, after the equation is reduced?

19. If twice John's age be added to three times Peter's age, the sum will be thirty-one years, and if twice Peter's be added to three times John's, the sum will be thirty-four. What is the age of each?

20. There are two numbers, such that three times the greater is one more than five times the less; and if twice the greater be added to three times the less, the sum will be twenty-six. What are the numbers?

21. A boy bought four oranges and three pears for fifteen cents; and again, at the same rate, he bought two oranges and five pears for eleven cents. What was the price of one of each?

22. The sum of two numbers is thirteen; and their difference is seven. What are the numbers?

23. If a slate and two writing-books cost twenty-five cents, at the same rate two slates would cost fifteen cents more than three writing-books. What was the cost of one of each?

24. There are two numbers, such that, if the greater be added to twice the less, the sum will be eleven; and if one be added to three times the less, the sum will be twice the greater. What are the numbers?

25. A man sold a cow and a pig, at such prices

that, if one fourth of the price of the cow be added to one half of the price of the pig, the sum will be eight dollars; and if the price of the pig be taken from five eighths of the price of the cow, the remainder will be eleven dollars. What was the price of each?

Let $x =$ the price of the cow,
and $y =$ the price of the pig.

(1.) By one condition of the question, . $\frac{x}{4} + \frac{y}{2} = 8.$

(2.) By another condition, ... $\frac{5x}{8} - y = 11$

(3.) Taking $\frac{y}{2}$ from each member of 1st, $\frac{x}{4} = 8 - \frac{y}{2}$

(4.) Adding y to each member of 2d, $\frac{5x}{8} = 11 + y$

(5.) Dividing each member of 4th by 5, $\frac{x}{8} = \frac{11+y}{5}$

(6.) Multiplying 5th by 2, and since $\frac{2x}{8} = \frac{x}{4}$, ... $\frac{x}{4} = \frac{22+2y}{5}$

(7.) Putting the value of $\frac{x}{4}$ in 3d and 6th equal to each other, $\frac{22+2y}{5} = 8 - \frac{y}{2}$.

(8.) Multiplying each member of 7th by 10, and reducing, . $44 + 4y = 80 - 5y.$

(9.) Adding $5y - 44$ to each member of 8th, ... $9y = 36$

(10.) Dividing 9th by 9, ... $y = 4.$

(11.) Putting 4, the value of y, for y, in 3d, ... $\frac{x}{4} = 8 - \frac{4}{2}$, or 6.

(12.) Multiplying 11th by 4, ... $x = 24$

The cow cost twenty-four dollars;
the pig cost four dollars.

26 There are two numbers, such that if two thirds

of the greater be added to two thirds of the less, the sum will be ten; and if one third of the less be added to three fourths of the greater, the sum will be ten. What are the numbers?

27. In the equation $\frac{2x}{3}+\frac{3y}{4}=12$, if $\frac{3y}{4}$ be subtracted from each member, what equation will represent the remainder?

28. If the equation $\frac{2x}{3}=12-\frac{3y}{4}$ be divided by 2, what equation will express the quotient?

29. If the equation $\frac{x}{3}=6-\frac{3y}{8}$ be multiplied by 3, what will express the product?

30. What must be done to the equation $\frac{2x}{3}+\frac{3y}{4}=12$, that the value of x may be found in terms of the equation? What expression will be equivalent to x?

31. If $x=18-\frac{9y}{8}$, what will $3x$ equal?

32. In the equation $\frac{2x}{3}+\frac{3y}{4}=12$, what expression will represent the value of y?

33. What will represent the value of $2y$, in the same equation?

34. What must be done to the equation $\frac{3x}{5}-\frac{y}{2}=7$, that an expression may be found equivalent to x?

35. What is the expression for the value of x, in the last equation?

36. Find equivalent expressions for the value of x, in the equations $\frac{2x}{3}+\frac{y}{4}=10$, and $\frac{3x}{4}-\frac{y}{2}=5$. What will the expressions be?

37. In the new equation formed by the values of x y is the only unknown quantity, and what is its value?

38. If you substitute, in the equation $\frac{3x}{4} - \frac{y}{2} = 5$, the value of y, as found above, in the place of y, what will be the value of x?

39. From the equations $\frac{x}{2} - \frac{y}{3} = 6$, and $\frac{x}{4} + \frac{y}{6} = 7$, make an equation which shall not contain the unknown quantity x. What will that equation be?

40. In the new equation thus formed, what will be the value of y?

41. If the value of y, thus found, be substituted for y, in the equation $\frac{x}{2} - \frac{y}{3} = 6$, what will be the value of x?

42. Two boys bought a dog for six dollars. John says, "I will give one half of my money, and you can give one third of yours, and that will just pay for him, but I shall own the greater part of him." "No," says Henry, "I will give two thirds of my money, and you shall give only one fourth of yours: that will pay for him, and I shall own more than you." How much money had each?

SECTION XXIX.

1. A MAN weighed two kinds of cannon balls of different weights. Three of the heavier and two of the lighter weighed twenty-one pounds, while one of

the heavier and one of the lighter weighed eight pounds. What was the weight of one of each kind?

Let $x =$ the weight of one of the heavier balls,
and $y =$ the weight of one of the lighter balls.

(1.) By one condition of the question, $3x + 2y = 21$.
(2.) By another condition, ... $x + y = 8$.
(3.) Taking x from each member of 2d, . $y = 8 - x$.
(4.) Multiplying 3d by 2, ... $2y = 16 - 2x$.
(5.) Putting the value of $2y$ for $2y$ in 1st, ... $3x + 16 - 2x = 21$.
(6.) Taking 16 from each member of 5th, and uniting terms, ... $x = 5$.
(7.) Putting 5, the value of x, in 3d, $y = 8 - 5$, or 3.

One kind of ball weighed 5 lbs;
the other kind weighed 3 lbs.

2. There are two numbers, such that three times the greater and twice the less, when added, are twenty-two, and twice the greater is six more than three times the less. What are the numbers?

Let $x =$ the greater number,
and $y =$ the less number.

(1.) By one condition of the question, $3x + 2y = 22$.
(2.) By another condition, ... $2x = 3y + 6$.
(3.) Dividing 2d by 2, ... $x = \frac{3y}{2} + 3$
(4.) Multiplying 3d by 3, ... $3x = \frac{9y}{2} + 9$.
(5.) Putting this value for $3x$ in 1st, ... $\frac{9y}{2} + 9 + 2y = 22$.
(6.) Multiplying 5th by 2, . . . $9y + 18 + 4y = 44$.
(7.) Taking 18 from each member of 6th, and uniting terms, ... $13y = 26$.

(8.) Dividing 7th by 13, ... $y = 2$

(9.) Putting 2, the value of y, in the 3d, ... $x = \frac{6}{2} + 3$, or 6

The greater number is 6;
the smaller is 2.

3. If, from each member of the equation $3x + 2y = 21$, $2y$ be taken, what equation will express the remainder?

4. If the equation $3x = 21 - 2y$ be divided by 3, what equation will express the quotient?

5. What must be done to the equation $3x + 2y = 21$, to find an expression equivalent to x, or that will represent the value of x?

6. If $x = 7 - \frac{2y}{3}$, what will $2x$ equal?

7. In the equation $3x + 2y = 21$, what expression will represent the value of y?

8. In the equation $4x + 3y = 22$, what expression will represent the value of x?

9. In the same equation, what expression will represent the value of y?

10. In the equation $2x + 3y = 12$, find an expression to represent the value of x. What will the expression be?

11. Substitute the value of x, thus found, in the place of x, in the equation $3x + 2y = 13$. What will the equation then be?

12. If the equation $18 - \frac{9y}{2} + 2y = 13$ be multiplied by 2, what equation will express the product?

13. In the equation $36 - 5y = 26$, if $5y$ be added to each member, what will express the result?

14. If 26 be taken from each member of the equation $36 = 26 + 5y$, what will express the result? and what is the value of y?

15. If, in the equation $2x + 3y = 12$, 2 be substituted in the place of y, what will the value of x be?

16. A man bought two barrels of cider and one of apples for eight dollars, and found that, at the same rate, three barrels of cider would cost five dollars more than two barrels of apples. What did a barrel of each cost?

17. There are two numbers, such that three times the greater is two more than four times the smaller, and five times the smaller is eight more than twice the greater. What are the numbers?

18. Twice John's money is twelve dollars more than Henry's; and twice Henry's is six dollars more than John's. How many dollars has each?

19. If you add four to the numerator of some fraction, the fraction will be one half; but if you add four to the denominator, the fraction will be only one fourth. What is the fraction?

Let $x =$ the numerator,
and $y =$ the denominator;
then $\frac{x}{y} =$ the fraction.

(1.) By one condition of the question, . . $\frac{x+4}{y} = \frac{1}{2}$

(2.) By another condition, ... $\frac{x}{y+4} = \frac{1}{4}$.

(3.) Multiplying 1st by y, ... $x + 4 = \frac{y}{2}$.

(4.) Multiplying 3d by 2, ... $2x + 8 = y$.

(5.) Multiplying 2d by 4, ... $\frac{4x}{y+4} =$

(6.) Multiplying 5th by $y+4$, . . . $4x = y+4$

(7.) Dividing 6th by 2, ... $2x = \frac{y}{2} + 2.$

(8.) Putting this value of $2x$ in the 4th, ... $\frac{y}{2} + 2 + 8 = y$

(9.) Subtracting $\frac{y}{2}$, and uniting terms in 8th, ... $10 = y - \frac{y}{2}.$

(10.) Multiplying by 2, ... $20 = 2y - y$, or y.

(11.) Subtracting 4 from each member of 3d, ... $x = \frac{y}{2} - 4.$

(12.) Putting 20, the value of y, in 11th, ... $x = \frac{20}{2} - 4$, or 6

6 = numerator,

20 = denominator;

the fraction is $\frac{6}{20}$.

20. Mary says one third of her age and four years more are equal to one half of Susan's age; and Susan says one fourth of her age and six years more are equal to five sixths of Mary's. How old is each?

21. If you subtract three from the numerator of some fraction, the fraction will be equal to one seventh; but if you subtract three from the denominator, the fraction will be equal to one. What is the fraction?

22. If the equation $\frac{2x}{3} = \frac{y}{4} + 4$ be multiplied by 3 and divided by 2, what equation will represent the result?

23. If 4 be taken from each member of the

equation $\frac{2x}{3} = \frac{y}{4} + 4$, and the remainder multiplied by 4, what expression will represent the value of y?

24. In the equation $\frac{x}{4} = \frac{2y}{3} - 2$ find the value of x, and substitute it for x, in the equation $2x = \frac{5y}{3} + 6$; and then find the respective values of x and y in numbers. What are they?

25. Samuel and Nathan have seven dollars. Half of Samuel's money is one dollar more than one third of Nathan's. How many dollars has each?

SECTION XXX.

1. When a number is multiplied by itself, the product is called the *second power* or *square* of that number, and the number itself is called the *second root* or *square root* of that square or product. Thus, $2 \times 2 = 4$; and 4 is the square or second power of 2, and 2 is the square root or second root of 4.

2. What is the square or second power of three?

3. What is the square root or second root of nine?

4. What is the second power or square of four?

5. What is the second root or square root of sixteen?

6. What is the second power or square of six?

7. What is the second or square root of forty-nine?

8. The square or second power of x is x times x, which may be expressed thus; $x\,x$, or x^2; and th

square root or second root of x^2 is x; or, using the *radical* sign, $\sqrt{x^2} = x$. The square of $3x$ is $9x^2$, and the square root of $9x^2$ is $3x$, or $\sqrt{9x^2} = 3x$.

9. What is the square or second power of y?

10. What is the square root or second root of y^2?

11. What is the square or second power of $2y$?

12. What is the second or square root of $4y^2$?

13. What is the square or second power of $5x$?

14. What is the second or square root of $36x^2$?

15. How many times is x contained in x?

16. If x be divided by x, what will the quotient be?

17. If 1 be multiplied by x, what will the product be?

18. If 2 be multiplied by x, what will the product be?

19. If 2 be multiplied by x^2, what will the product be?

20. If $2x$ be divided by x, what will be the quotient?

21. If $2x$ be divided by 2, what will the quotient be?

22. If $7x$ be divided by x, what will the quotient be?

23. If $4x$ be multiplied by 2, what will the product be?

24. If $8x$ be divided by $4x$, what will be the quotient?

25. If $2x$ be multiplied by x, what will be the product?

26. If $2x$ be multiplied by $2x$, what will the product be?

27. If $4x^2$ be divided by $2x$, what will the quotient be?

28. If $4x^2$ be divided by $4x$, what will the quotient be?

29. What is the second power of $3x$?

30. What is the square root of $9x^2$?

31. If $x = 3$, to what will x^2 be equal?

32. If $x^2 = 9$, to what will x be equal?

33. If $x = 4$, to what will x^2 be equal?

34. If $x^2 = 16$, to what will x be equal?

35. If $x = 5$, to what will x^2 be equal?

36. If $x^2 = 36$, what will x equal?

37. If $2x^2 = 18$, what will x^2 equal?

38. If the equation $2x^2 = 18$ be divided by 2, what equation will represent the quotient?

39. If the equation $3x^2 = 12$ be divided by 3, what equation will express the quotient? What will be the value of x?

40. Find the square root of each member in the equation $x^2 = 16$. What is the value of x?

41. Find the square root of each member of the equation $4x^2 = 36$. What will be the value of x?

42. The square of $\frac{2x}{3}$ is $\frac{2x}{3}$ multiplied by $\frac{2x}{3}$, which is $\frac{4x^2}{9}$; and the square root of $\frac{4x^2}{9}$ must therefore be $\frac{2x}{3}$. What is the square of $\frac{x}{2}$?

43. What is the square root of $\frac{x^2}{4}$?

44. What is the square of $\frac{x}{3}$?

45. If $\frac{x}{3}$ be multiplied by $\frac{x}{3}$, what will be the product?

46. What is the square root of $\frac{4x^2}{25}$?

47. What is the square of $\frac{x}{5}$?

48. If $\frac{2x}{5}$ be multiplied by $\frac{2x}{5}$, what will be the product?

49. What is the second root of $\frac{25x^2}{64}$?

50. To what is the expression $\sqrt{\frac{16x^2}{25}}$ equal?

SECTION XXXI.

1. John says, if his age were multiplied by his age, the product would be five times his age. How old is he?

Let x represent John's age;

then $x \times x = x^2$, the product of his age multiplied by his age.

But this product must be equal to 5 times his age or $5x$;

therefore, by a condition of the question,

$$x^2 = 5x.$$

Dividing this equation by x, gives

$$x = 5;$$

for, if $x \times x$ is equal to 5 *times* x, or x *times* 5, then, x *not multiplied by* x, must be equal to 5 *not multiplied by* x;

and the age of John is 5 years.

2. What number multiplied by itself will be six times the number?

3. A father is six times as old as his son, and his age is the square of his son's age. What is his son's age?

4. The square or second power of a number is seven times the number. What is the number?

5. George has twice as many cents as Robert, and if Robert's money be multiplied by the number of cents that George has, the product will be double the money that both have. How many cents has each?

Let $x =$ Robert's money;

then $2x =$ George's money,

and $x + 2x = 3x$ will be the sum of money that both have,

and $x \times 2x = 2x^2$ will be the product of one's money by the other's.

Therefore, by the conditions of the question,

$$2x^2 = 6x.$$

Dividing this by $2x$, gives

$$x = 3.$$

$2x = 2 \times 3$, or 6;

then George has 6 cents, and Robert 3 cents.

6. There is a field which is as many rods long as it is broad; and twice the product of its length by its breadth is equal to ten times its length. What is the length of one side of the field?

7. Four times the product of a number multiplied

by itself is equal to sixteen times the number. What is the number?

8. Sarah has three times as many books as Jane. If Sarah's number of books be multiplied by Jane's number, the product will be nine times the difference between Sarah's and Jane's. How many has each?

9. One number is twice as large as another; and the product of the two numbers is fourteen times their difference? What are the numbers?

10. There is a square field, and its length multiplied by its breadth, or the number of square rods in the field, is equal to the number of rods round it. How many rods long is one side of the field?

11. One number is three times another; and the product of the two is equal to three times their sum. What are the numbers?

12. John has four times as many apples as William; and if John's number of apples be multiplied by twice William's number, the product will be eight times what they both had. How many had each?

13. One number is twice another; and if three times the smaller be multiplied by twice the larger, the product will be twenty-four times their difference. What are the numbers?

14. If 2 be multiplied by x, what will be the product?

15. If $2x$ be divided by x, what will be the quotient?

16. If $2x$ be divided by 2, what will be the quotient?

17. If $6x$ be divided by $3x$, what will be the quotient?

18. If $4x^2$ be divided by x, what will be the quotient?

19. If the equation $x = 4$ be multiplied by x, what equation will express the product?

20. If $4x^2$ be divided by $4x$, what will be the quotient?

21. If $x^2 = 4x$, what will x equal?

22. If the equation $x = 4$ be multiplied by $2x$, what will express the product?

23. If the equation $x^2 = 4x$ be divided by x, what equation will express the quotient?

24. If the equation $2x^2 = 8x$ be divided by $2x$, what equation will express the quotient?

25. If $5x^2 = 15x$ be divided by $5x$, what equation will express the quotient?

26. In the equation $4x^2 = 12x$, what number does x represent?

27. Reduce the equation $6x^2 = 18x$, and find the value of x.

28. A father is five times as old as his son, and the product of their ages is five times the sum of their ages. What are their ages?

29. One number is four times another, and twelve times their sum is equal to three times their product. What are the numbers?

30. A field is three times as many rods long as it is wide, and the product of the length by twice the width is eighteen times the difference between the length and width. How many rods long is each side?

31. One number is five times another, and their product is equal to ten times their difference. What are the numbers?

32. A farmer has twice as many oxen as horses; and if the number of oxen be multiplied by the number of horses, the product will be twice the sum of the oxen and horses together. How many of each has he?

SECTION XXXII.

1. A BOY, being asked his age, replied, "If my age were multiplied by one half of my age, the product would be six times my age." How old was he?

Let $x =$ his age;

then $\frac{x}{2} =$ one half of his age,

and $x \times \frac{x}{2} = \frac{x^2}{2}$ will be his age multiplied by half of his age.

By the conditions of the question,

$$\frac{x^2}{2} = 6x.$$

Multiplying each member by 2,

$$x^2 = 12x.$$

Dividing each member by x,

$$x = 12;$$

therefore the boy's age was twelve years.

2. What number, multiplied by one third of itself, will give a product equal to twice the number?

3. If $\frac{x^2}{2}$ be multiplied by 2, what will be the product?

4. If x^2 be divided by 2, what will be the quotient?

5. What expression will represent one half of x^2?

6. If x^2 be divided by 3, what will express the quotient?

7. What expression will represent one fourth of x^2?

8. What expression will represent three fourths of x^2?

9. If $2x^2$ be divided by 3, what expression will represent the quotient?

10. If $3x^2$ be divided by 4, what will express the quotient?

11. If $3x^2$ be divided by 2, what will express the quotient?

12. If the equation $x^2 = 4$ be divided by 2, what equation will represent the quotient?

13. If the equation $x^2 = 6x$ be divided by 3, what will represent the quotient?

14. If the equation $2x^2 = 12x$ be divided by 3, what will express the quotient?

15. What is one third of the equation $2x^2 = 12x$?

16. What is two thirds of the equation $x^2 = 6x$?

17. If the equation $\frac{x^2}{3} = 3x$ be multiplied by 3, what equation will represent the product?

18 If the equation $\frac{2x^2}{3} = 6x$ be divided by 2, what equation will express the quotient?

19. In the equation $\frac{3x^2}{5} = 3x$, what is the value of x?

20. If one fourth of George's money be multiplied by the whole of his money, the product will be four times his money. How many dollars has he?

21. One number is one fifth of another, and their

product is equal to twice the greater. What are the numbers?

22. A man, being asked how many children he had, replied, that if the number of his children were multiplied by itself, three fifths of the product would be six times the number of his children. How many had he?

23. If the square of a number be added to one fourth of the square, the sum will be ten times the number. What is the number?

24. Caroline has one third as many books as Eliza, and the difference between the square of Eliza's number and the square of Caroline's number is twelve times the number that Eliza has more than Caroline. How many has each?

25. One number is one half of another, and the sum of their squares is ten times the sum of the numbers. What are the numbers?

26. A farmer bought a cow for one half of what he gave for a horse. If the square of the cost of the cow be subtracted from the square of the cost of the horse, the difference will be twenty-five times the cost of both. What did he pay for each?

27. One number is one fifth of another, and the square of the smaller is equal to the difference between the two numbers. What are the numbers?

28. A received twice as many dollars as B. If A's money be multiplied by two thirds of B's, the product will be four times the sum of what they both received. How much money did each receive?

29 One number is two thirds of another, and

their product is three times the less. What are the numbers?

30. A bridle cost one fourth as many dollars as a saddle; and if the price of the bridle be multiplied by the price of the saddle, the product will be the price of four saddles. What was the cost of each?

31. What is the square of $2x$?

32. What is the square of $\frac{x}{3}$?

33. What is the square of $\frac{2x}{3}$?

34. What is the square of $\frac{3x}{5}$?

35. If x^2 be added to $\frac{x^2}{2}$, what will express the sum in one term?

36. Reduce the expression $x^2 + \frac{2x^2}{3}$ to one term.

37. If $\frac{x^2}{2}$ be taken from x^2, what will express the remainder?

38. If $\frac{2x^2}{3}$ be taken from x^2, what will express the remainder?

39. If $\frac{x^2}{4}$ be taken from $\frac{3x^2}{4}$, what will represent the remainder?

40. If $\frac{2x^2}{5}$ be subtracted from x^2, what will express the remainder?

41. If the equation $\frac{x^2}{3} = x$ be multiplied by 3, what equation will represent the product?

42. If the equation $\frac{3x^2}{4} = 6x$ be multiplied by 4,

and that product divided by $3x$, what equation will express the result?

43. What must be done to the equation $\frac{2x^2}{3} = 2x$ to find the value of x? What number does x represent?

44. Reduce the equation $\frac{2x^2}{5} = 4x$. What will be the value of x?

SECTION XXXIII.

1. A BOY, being asked his age, said, if his age were multiplied by itself, the product would be forty-nine. What was his age?

Let $x =$ the boy's age;

then $x \times x = x^2$ will be his age multiplied by itself.

But his age multiplied by itself is 49;

therefore, by the conditions of the question,

$$x^2 = 49.$$

Extracting the square root of each member, that is, finding a quantity which, multiplied by itself, will produce each member, gives

$$x = 7, \text{ the boy's age.}$$

It is evident that x is the second root of x^2, because

$$x \times x = x^2;$$

and 7 is the second root of 49, because

$$7 \times 7 = 49.$$

2. The second power or square of a number is nine. What is the number?

3. If x^2 be divided by x, what will be the quotient?

4. If forty-nine be divided by seven, what will be the quotient?

5. If the square root of the equation $x^2 = 49$ be extracted, what equation will express the result?

6. If each member of the equation $x = 7$ be multiplied by itself, what equation will express the product?

7. If the equation $2x^2 = 18$ be divided by 2, what will express the quotient? and what will be the value of x?

8. John has twice as much money as George, and the product of George's money multiplied by John's is fifty cents. How many cents has each?

9. Three times the product of a number by itself is forty-eight. What is the number?

10. A man received as many shillings a day as he worked days. How many days did he work to obtain six dollars?

11. Four times the square or second power of a number is one hundred. What is the number?

12. There is a square field containing one hundred and twenty-one square rods. How long is one side of the field?

13. One half of the product of a number multiplied by itself is eight. What is the number?

14. If each member of the equation $\frac{x^2}{2} = 8$ be multiplied by 2, what equation will express the product?

15. If each member of the equation $\frac{2x^2}{3} = 24$ be

multiplied by 3, what equation will express the product?

16. If each member of the equation $2x^2 = 72$ be divided by 2, what will express the quotient?

17. In the equation $\frac{2x^2}{3} = 24$, what number is represented by x?

18. In the equation $\frac{3x^2}{5} = 60$, what is the value of x?

19. Sarah's age is one half of Matilda's, and the product of their ages is thirty-two. What is the age of each?

20. If one half of a number be multiplied by one third of the same number, the product will be twenty-four. What is the number?

21. A field containing ninety-six square rods is two thirds as wide as it is long. What is its length, and what its width?

22. If one half of a number be added to itself, and the sum multiplied by itself, the product will be ninety-six. What is the number?

23. If $\frac{5x^2}{2} = 40$ be multiplied by 2, what equation will express the product?

24. If $5x^2 = 80$ be divided by 5, what will be the quotient?

25. In the equation $\frac{5x^2}{2} = 40$, what is the value of x?

26. A farmer said, if one half of his sheep were multiplied by one fourth of them, the product would be fifty. How many had he?

27. If six be added to the square of a number, the sum will be fifty-five. What is the number?

28. If 3 be subtracted from each member of the equation $x^2 + 3 = 19$, what equation will express the remainder? and what will be the value of x?

29. If 7 be added to each member of the equation $2x^2 - 7 = 43$, what will express the sum? and what will be the value of x?

30. A boy said, if two cents were added to twice the square of his money, he would have one dollar How many cents had he?

31. The product of two numbers is sixteen, and the less is contained four times in the greater. What are the numbers?

32. A boy paid eighty-one cents for melons, giving for each melon a number of cents equal to the number of melons that he bought. How many did he buy?

33. If the equation $\frac{x^2}{6} = \frac{2}{3}$ be multiplied by 6, what equation will represent the product? and what will be the value of x?

34. Since the square root of four is two, and the square root of nine is three, what will be the square root of four ninths?

35. What is the square root of sixteen twenty-fifths?

36. What is the value of x in the equation $x^2 = \frac{25}{49}$?

37. What is the value of x in the equation $2x^2 = \frac{72}{81}$?

38. What is the value of x in the equation $\frac{x^2}{2} = \frac{8}{25}$?

SECTION XXXIV.

1. If x times x is x^2, and x times 3 is $3x$, what expression will represent x times $\overline{x+3}$?*

2 If the expression $x+4$ be multiplied by x, what will represent the product?

3 If the expression $x+9$ be multiplied by x, what will be the product?

4. If the expression $x+3$ be multiplied by 3, what will express the product?

5. When the expression $x+3$ is multiplied by x, the product is x^2+3x; and when $x+3$ is multiplied by 3, the product is $3x+9$. What will represent the sum of their products?

6. If $\overline{x+3}$ be multiplied by $\overline{x+3}$, what will express the product? that is, what will be the second power of $x+3$?

First, multiplying $\overline{x+3}$ by x;

x times x, or $x \times x = x^2$,

and x times 3, or $x \times 3 = 3x$;

then their sum is x^2+3x.

Next, multiplying $\overline{x+3}$ by 3;

3 times x, or $3 \times x = 3x$,

and 3 times 3, or $3 \times 3 = 9$;

then their sum is $3x+9$.

Thus, $3x+9$, added to x^2+3x, is

$$x^2+3x+3x+9.$$

Uniting terms, gives x^2+6x+9,

which is the square or second power of $\overline{x+3}$.

* A *bar* ——— or *parenthesis* ( ) embraces terms to be taken together, or subject to the same operation

Or multiplying $x+3$
by $x+3$,

x^2+3x, x times $\overline{x+3}$.
$3x+9$, 3 times $\overline{x+3}$.

Sum of products, x^2+6x+9, or square of $x+3$.

It is evident, from inspecting the above, that the square of $x+3$ is x^2, twice $x \times 3$, and the square of three; that is, *the square of the first term, and twice the first term multiplied by the last, and the square of the last term.*

7. What is the square root of the expression x^2+6x+9?

Extracting the square root of x^2, the first term of the square, gives x for the first term of the root.

Since $6x$ is *twice the first term* of the root multiplied by the last, that is, *twice* x multiplied by the last term of the root, if $6x$ be divided by twice x, the quotient, 3, will be the last term of the root.

Therefore, $x+3$ must be the square root of the expression x^2+6x+9. See 6th.

8. In the expression x^2+6x, what term is wanting to make the expression a perfect square?

Extracting the square root of x^2, the first term of the expression, gives x for the first term of the root.

Since $6x$ is *twice the product of the two terms* of the root, if $6x$ be divided by $2x$, that is, by *twice the first term* of the root, the quotient, 3, will be the *last term* of the root.

As 3 is the other term of the root, 9, its square, must be added to the expression x^2+6x, to complete

the square, that is, to make the expression a perfect square.

Then the expression $x^2 + 6x + 9$, is a perfect square

9. What must be added to an expression, consisting of two terms, only one of which is a second power, to complete the square?

Extract the square root of the term containing the second power, and divide the other term by twice the square root thus found; the quotient will be the square root of the term that must be added to the expression to make it a perfect square. This is deduced from the preceding.

10. To find the square root of a perfect second power, consisting of three terms.

Extract the square root of the first term, which is a perfect square, for the first term of the root.

Since the term, which is not a perfect square, is twice the first term of the root multiplied by the last term of the root, divide this term of the square by twice the term of the root already found, and the quotient will be the other term of the root. This is evident from inspection of the 6th and 7th, and explains the rule for extraction in arithmetic.

11. What is the second power of $x + 4$?

By the 6th, the square of the first term is $x \times x = x^2$.

Twice the product of both terms is $2 \times x \times 4 = 8x$.

The square of the last term is $4 \times 4 = 16$.

Adding these three results, gives $x^2 + 8x + 16$.

12. What must be added to the expression $x^2 + 8x$, to make it a perfect square?

It is evident that the square of the second term of the root must be added.

To find the second term of the root, divide $8x$ by twice the first term of the root, that is, by $2x$, which is twice the root of x^2, and the quotient, 4, will be the second term of the root; therefore, 16, the square of 4, must be added to $x^2 + 8x$ to complete the square.

13. What is the second power of $x + 5$?

14. What is the second or square root of $x^2 + 10x + 25$?

15. What must be added to the expression $x^2 + 10x$, to make it a perfect second power?

16. What is the second power of $x + 6$?

17. What is the second root of $x^2 + 12x + 36$?

18 What must be added to the expression $x^2 + 12x$, to make it a perfect square?

19. What is the second power or square of $5 + 3$?

By the 6th, the square of the first term is $5 \times 5 = 25$.

Twice the first term by the last is $2 \times 5 \times 3 = 30$

The square of the last term is $3 \times 3 = 9$.

Collecting the products, gives $25 + 30 + 9$

Or, multiplying $5 + 3$

by $5 + 3$

$25 + 15$, or 5 times $\overline{5+3}$,

$15 + 9$, or 3 times $\overline{5+3}$.

Sum of products, $25 + 30 + 9$.

But $25 + 30 + 9 = 64$, $5 + 3 = 8$, and $8 \times 8 = 64$.

Then the second power of $5 + 3$ consists of the square of $5 = 25$, twice $5 \times 3 = 30$, and the square of $3 = 9$;

or, the square of the first term, twice the product of the two terms, and the square of the last term; as in 6th.

20. What must be added to $25 + 30 = 55$, to make the sum a perfect square?

It is evident from the 8th and 9th, that the square of the last term of the root must be added.

To find the last term of the root, divide twice the product of the two terms, which is 30, by twice the square root of 25, which is 2×5, and the quotient, 3, will be the last term of the root.

Then 9, the square of 3, must be added to $25 + 30$, giving $25 + 30 + 9 = 64$, a perfect square.

21. What is the square root of $25 + 30 + 9$?

Extract the square root of 25, which is 5. Since 30 is twice the first term of the root, multiplied by the last term of the root, if 30 be divided by twice 5, or 10, the quotient, 3, will be the last term of the root; therefore, the root is $5 + 3$.

Or, since $25 + 30 + 9 = 64$, the square root of the equation is $5 + 3 = 8$.

22. What is the second power of $10 + 2$?

23. What is the second root of $100 + 40 + 4$?

24. What must be added to $100 + 40$, to make the sum a perfect square?

25. What is the square root of $100 + 100 + 25$?

26. What is the second power of $2x + 3$?

27. What is the square root of $4x^2 + 12x + 9$?

The square root of $4x^2$ is $2x$; and the quotient of

$12x$ divided by twice $2x$ is 3; therefore, the square root is $2x + 3$.

28. What must be added to the expression $4x^2 + 12x$, to make the expression a perfect square?

$12x$ divided by twice $2x$, or twice the first term of the root, is 3, the second term of the root; therefore 9, the square of 3, must be added.

29. What is the square of $3x + 4$?

30. What is the square root of $9x^2 + 24x + 16$?

31. What must be added to $9x^2 + 24x$, to make the expression a perfect square?

32. What must be added to $x^2 + x$, to make the expression a perfect square?

It is evident that the square must be so completed that x shall be twice the product of the two terms of the root; then, if x be divided by twice the first term of the root, the quotient will be the second term.

But x divided by $2x$, thus $\frac{x}{2x}$, is equal to one half; therefore $\frac{1}{2}$ is the second term of the root, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ is the square of the second term; therefore $\frac{1}{4}$ must be added to the expression,

and $x^2 + x + \frac{1}{4}$ is a perfect second power.

33. What is the product of $\overline{x + \frac{1}{3}}$ multiplied by $\overline{x + \frac{1}{3}}$?

Multiplying $\overline{x + \frac{1}{3}}$ by x, gives $x^2 + \frac{x}{3}$.

Multiplying $\overline{x + \frac{1}{3}}$ by $\frac{1}{3}$ gives $\frac{x}{3} + \frac{1}{9}$.

The products added are $x^2 + \frac{2x}{3} + \frac{1}{9}$.

Multiplying $x+\frac{1}{3}$
by $x+\frac{1}{3}$

is $x^2+\frac{x}{3}$, or x times $\overline{x+\frac{1}{3}}$,

and $\frac{x}{3}+\frac{1}{9}$, or $\frac{1}{3}$ of $\overline{x+\frac{1}{3}}$.

Sum of products is $x^2+\frac{2x}{3}+\frac{1}{9}$, the square of $\overline{x+\frac{1}{3}}$

34. What will express the square of $\overline{x+\frac{1}{2}}$?

35. What must be added to $x^2+\frac{2x}{3}$, to complete the square?

36. What is the square root of $x^2+\frac{2x}{3}+\frac{1}{9}$?

37. What is the square root of $x^2+x+\frac{1}{4}$?

38. What is the second power of $\overline{x+\frac{1}{4}}$?

39. What must be added to $x^2+\frac{x}{2}$, to make the expression a complete square?

40. What is the square root of $x^2+\frac{x}{2}+\frac{1}{16}$?

41. What must be added to $x^2+\frac{4x}{3}$, to complete the square?

42. What is the second power of $\overline{x+\frac{2}{3}}$?

43. What is the second root of $x^2+\frac{4x}{3}+\frac{4}{9}$?

44. What is the square root of $x^2+\frac{3x}{2}+\frac{9}{16}$?

45. What is the second power of $\overline{x+1\frac{1}{2}}$, or $\overline{x+\frac{3}{2}}$?

46. What must be added to $x^2+\frac{3x}{2}$, to complete the square?

47. What must be added to x^2+3x, to complete the square?

48. What is the second power of $x + \frac{1}{5}$?

49 What is the second root of $x^2 + \frac{8x}{5} + \frac{16}{25}$?

SECTION XXXV.

1. A LADY says, "If twice my son's age and one year more, are added to the square of my son's age, the sum will be eighty-one years." How old is her son?

Let $x =$ the son's age.

By the conditions of the question, $x^2 + 2x + 1 = 81$

Extracting square root, $x + 1 = 9$.

The square root of the first member is $x + 1$, because $\overline{x+1}$ multiplied by $\overline{x+1}$, is equal to $x^2 + 2x + 1$.

The square root of 2d member is nine, because

$9 \times 9 = 81$.

Taking 1 from each member $x = 8$.

The son's age is 8 years.

2. If four times a number be added to its square, and four more added to the sum, the whole will be sixty-four. What is the number?

Let $x =$ the number.

By the conditions of the question, $x^2 + 4x + 4 = 64$.

The square root of each member must be found.

The square root of 64 is 8, because $8 \times 8 = 64$.

Now, the square root of the first term, in the expression $x^2 + 4x + 4$, is x, because $x \times x = x^2$.

The remaining terms of the expression, that is, $4x$

$+4$, consist of *twice the first term* of the root, that is, $2x$, multiplied by the *last term* of the root, also the *square* of the *last term* of the root.

If $4x$, or twice the first term of the root multiplied by the last term of the root, be divided by $2x$, or twice the first term of the root, the quotient will be 2, or the other term of the root; because $2x+2$ multiplied by the last term, 2, gives $4x+4$, the remaining part of the expression x^2+4x+4;

and $\overline{x+2}$, or the square root of the expression, multiplied by $\overline{x+2}$, equals x^2+4x+4.

Since $x+2$ is the second root of the first member, it must equal 8, the second root of the other member;

therefore, $x+2=8$.

Taking 2 from each member, $x=6$, *Ans.*

3. If six times a man's money, and nine dollars more, be added to the second power of his money, the sum will be one hundred dollars. How much money has he?

4. If the square of a number be added to eight times the number, the sum will be twenty. What is the number?

Let $x=$ the number.

By the conditions of the question, $x^2+8x=20$.

In this equation, as neither member is a perfect square, the square of the last term of the root must be found and added to each member; then each will be a perfect second power.

If $8x$, or twice the product of both terms of the root, be divided by $2x$, or twice the first term, the quotient will be 4, or the last term of the root;

therefore 16, the square of 4, must be added to each

member of the equation, to complete the square of each.

Adding 16 to each, $x^2 + 8x + 16 = 20 + 16 = 36$.

Extracting the square root, $x + 4 = 6$.

Taking 4 from each member, $x = 2$, *Ans.*

5. A man travelled as many hours as he travelled miles in one hour. If the whole distance that he travelled be added to four times the distance that he went in one hour, the sum will be seventy-seven miles. How many miles did he travel in an hour? and what was the whole distance?

6. Complete the square in the equation $x^2 + 16x = 57$. What will be the value of x?

7. What must be added to each member of the equation $x^2 + 14x = 15$, to make each a perfect square? What will be the value of x?

8. If the equation $4x^2 + 16x = 84$ be divided by 4, what will express the result? What must be added to each member of the quotient to complete the square? and what is the value of x?

9. What must be added to each member of the equation $4x^2 + 4x = 80$, to complete the square? and what is the value of x?

The root of $4x^2$ is $2x$, and twice $2x$ is contained in $4x$ once; therefore, the last term of the root is 1, and its square, or 1, must be added to each member, making $4x^2 + 4x + 1 = 81$.

10. What must be added to each member of the equation $9x^2 + 12x = 21$, to complete the square? and what is the value of x?

11 What must be added to the equation $16x^2 +$

$24\,x = 112$, to make each member a perfect second power? and what is the value of x?

12. A boy has a number of pencils in each hand, and four more in the right hand than he has in the left. If the number in his right hand be multiplied by the number in his left hand, the product will be forty-five. How many has he?

13. One number is four more than another, and if four times the smaller be multiplied by the larger, the product will be forty-eight. What are the numbers?

14. A man, in buying sheep, gave for each one a number of dollars equal to the number of sheep that he bought. If he had purchased four times as many as he did, and eight more, at the same rate, they would have cost sixty dollars. How many did he buy? and at what price?

15. If twenty-five times the square of a number be added to fifty times the number, the sum will be two hundred. What is the number?

16. A man spent part of his money, lost four dollars more than he spent, and then found his purse empty. If what he lost be multiplied by what he spent, the product will be ninety-six. How much did he spend? and how many dollars had he at first?

17. If to the square of a number one half of the number be added, the sum will be five. What is the number?

Let $x =$ the number.

By conditions of the question, $x^2 + \frac{x}{2} = 5$.

$\frac{x}{2}$ divided by $2\,x$, gives $\frac{1}{4}$, the 2d term of the root

Adding $\frac{1}{16}$, the square of $\frac{1}{4}$, $x^2 + \frac{x}{2} + \frac{1}{16} = 5 + \frac{1}{16}$, or $\frac{8}{16}$.

Extracting square root, $x + \frac{1}{4} = \frac{9}{4}$

Reduced, $x = 2$, *Ans.*

18. What must be added to each member of the equation $x^2 + \frac{2x}{3} = 11$, to make it a perfect square? and what will be the value of x?

19. What must be added to each member of the equation $x^2 + \frac{4x}{5} = 5\frac{3}{5}$, or $\frac{28}{5}$, to make it a perfect second power? and what is the value of x?

Completing square, $x^2 + \frac{4x}{5} + \frac{4}{25} = \frac{28}{5} + \frac{4}{25} = \frac{144}{25}$.

Square root extracted, $x + \frac{2}{5} = \frac{12}{5}$. Reduced, $x = 2$.

Ans. $\frac{4}{25}$ must be added, and the number is 2.

20. What must be added to each member of the equation $x^2 + \frac{x}{4} = \frac{5}{4}$, to complete the square, and find the value of x?

21. In the equation $x^2 + \frac{x}{2} = \frac{1}{2}$, what is the value of x?

22. What must be added to $\frac{x^2}{9} + \frac{x}{3} = 6$, to find the value of x?

The square root of $\frac{x^2}{9}$ is $\frac{x}{3}$; and twice that root, or $\frac{2x}{3}$, is contained in $\frac{x}{3}$, the middle term, one half of a time; therefore, $\frac{1}{2}$ is the other term of the root, and $\frac{1}{4}$, or its square, must be added to each member, making $\frac{x^2}{9} + \frac{x}{3} + \frac{1}{4} = 6\frac{1}{4} = \frac{25}{4}$

Square root extracted, $\frac{x}{3} + \frac{1}{2} = \frac{5}{2}$.

Reduced, $x = 6$, *Ans.*

23. A man, being asked how much money he had, said, if half of his money were multiplied by half of his money, and the product added to half of his money, the sum would be thirty dollars. How much had he?

24. One number is three more than another, and if one fourth of the greater be multiplied by the less, the product will be one. What are the numbers?

25. The length of a room is four feet more than its breadth. If one half the length be multiplied by half the breadth, the product will be forty-eight feet. What is the size of the room?

26. If one fourth of a number be added to the square of a number, the sum will be three eighths. What is the number?

27. A, being asked what part of a ship he owned, replied, if one half of the ship were added to one half of his share, and the sum were multiplied by one half of his share, the product would be three sixteenths of the ship. What part did he own?

SECTION XXXVI.

1. What is the product of $\overline{x-1}$ multiplied by x?

2. What is the product of $\overline{x-1}$ multiplied by 1?

3. If $x^2 - x$ be added to $x - 1$, what will be the sum?

4. If $x-1$ be multiplied by $x+1$, what expression will represent their product?

5. If $x+1$ be multiplied by $x-1$, will the product be the same as in the preceding question?

x times $\overline{x+1}$ is x^2+x; but this is *once* $\overline{x+1}$ too many; therefore, $x+1$ must be taken from x^2+x; then $x^2+x-x-1=x^2-1$, the same as above.

6. Multiply $x+2$ by $x-2$. What will be the product?

The product will be x times $\overline{x+2}$, *less* 2 times $\overline{x+2}$.

x times $\overline{x+2}$ is x^2+2x;

less 2 times $\overline{x+2}$ is $-2x-4$.

Sum of products is . . . x^2-4.

Or, multiplying $x+2$

by $x-2$,

gives x^2+2x,

and $-2x-4$.

Sum of products is x^2-4, as above.

Hence, if only one of two factors has the sign $-$ before it, the product must have the same.

7. What is the product of $\overline{x+3}\times\overline{x-3}$?

8. What is the product of $\overline{x+4}\times\overline{x-4}$?

9. What is the product of $(x+4)\times(x-5)$?

The product will be x times $\overline{x+4}$, *less* 5 times $\overline{x+4}$.

x times $x+4$ is x^2+4x;

-5 times $x+4$ is $-5x-20$.

Sum of products is x^2-x-20.

10. What is the product of $(x+5)\times(x-7)$?

11. What is the product of $(x+5)\times(x-4)$?

12. What is the product of $\overline{x+6} \times \overline{x-6}$?

13. What is the product of $\overline{x-6} \times \overline{x+5}$?

14. What is the product of $\overline{x+6} \times \overline{x-5}$?

15. What is the product of $\overline{x+7} \times \overline{x-7}$?

16. What is the product of $(x+7) \times (x-2)$?

17. What is the product of $\overline{x+2} \times \overline{x-7}$?

18. What is the product of $\overline{x+8} \times \overline{x-8}$?

19. What is the product of $(x-10) \times (x+10)$?

20. If $\overline{x-2}$ be taken from $\overline{x+2}$, what will express the difference? *Vide* Sect. XIX.

21. If $\overline{x+2} \times \overline{x+2}$, what will be the product?

22. If $\overline{x+2} \times \overline{x-2}$, what will be the product?

23. If this last product (x^2-4) be taken from the preceding product (x^2+4x+4), what will express the difference?

$4x+8$, the difference in the last, is 4 times $\overline{x+2}$, the multiplicand. Then the difference between the multipliers is 4, as found in question 20.

24. What is the product of $\overline{x-1} \times x$?

25. If $(x-1)$ be taken from (x^2-x), what expression will represent the remainder?

It is evident, if the whole of x be taken from x^2-x, that x^2-2x, the remainder, would be too small by one; because not the whole of x is to be taken away, but x *less* 1; one must then be added to x^2-2x, making x^2-2x+1.

It is also evident that, to subtract a term, the sign before it, if *plus*, must be changed to *minus*, and if *minus*, to *plus*.

26. What will express the product of $\overline{x-1} \times \overline{x-1}$?

x times $\overline{x-1}$ will be *once* $x-1$ too many; therefore, $x-1$ must be taken from x times $\overline{x-1}$; $\overline{x-1} \times x$ is x^2-x; less once $\overline{x-1}$ is x taken away and 1 added; expressed thus, $x^2-x-x+1=x^2-2x+1$.

Thus we see that $\overline{x-1} \times -1$, becomes $-x+1$.

Or, multiplying $x-1$
by $x-1$,
gives $\overline{x^2-x}$,
and $\quad -x+1$.

Sum of products is $\overline{x^2-2x+1}$.

Thus it is apparent, that a *plus* term multiplied by a *plus* term gives a *plus* term for the product, and a *minus* term multiplied by a *minus* term gives a *plus* term. A *minus* term by a *plus* term, or a *plus* term by a *minus* term, gives a *minus* term for the product.

27. What is the second power of $x-1$?

As it is $\overline{x-1} \times \overline{x-1}$, it will be seen, by inspecting the preceding, that the square or second power of $x-1$ or $\overline{x-1}^2$ is the square of the first term, x, which is x^2, added to the square of the last term -1, which is $+1$; and twice the first term, x, multiplied by the last term -1, which is $-2x$; connecting terms x^2-2x+1.

28. What is the second power of $x-2$, or $(x-2)^2$?

The square of x is x^2; twice $x \times -2$ is $-4x$; and

the square of -2 is $-2 \times -2 = 4$. Sum of products is $x^2 - 4x + 4$.

29. What is the second power of $\overline{x-3}$, or $\overline{x-3}^2$?

30. What is the second or square root of $x^2 - 2x + 1$, or $\sqrt{x^2 - 2x + 1}$ is equal to what?

This may be found by extracting the root of the first term, and dividing the middle term by twice that root, because the middle term is twice the product of the two terms.

The square root of x^2 is x; twice this root, or $2x$, is contained in $-2x$, which is twice the first term $\times$ by the last, -1 time, because $2x$ multiplied by -1 is $-2x$; therefore, the root is $x - 1$.

31. What must be added to the expression $x^2 - 2x$, to make the expression a perfect square?

32. What must be added to $x^2 - 4x$, to make the expression a perfect square?

33. What is the square root of $x^2 - 4x + 4$?

34. What must be added to $x^2 - 6x$, to make the expression a perfect square?

35. What is the square root of $x^2 - 6x + 9$?

36. What is the second power of $x - 4$?

37. What must be added to $x^2 - 8x$, to make the expression a perfect square?

38. What is the second root of $x^2 - 8x + 16$?

39. What is the second power of $x - 5$?

40. What must be added to $x^2 - 10x$, to complete the square?

41. What is the second root of $x^2 - 10x + 25$?

42. What is the second power of $x - \frac{1}{2}$?

The square of the first term, x, is x^2; twice the fir

term, x, $\times$ by the last, $-\frac{1}{2}$, is $-x$, and the square of the last term $-\frac{1}{2}$ is $+\frac{1}{4}$. *Ans.* $x^2 - x + \frac{1}{4}$.

Or, multiplying $x - \frac{1}{2}$

by $x - \frac{1}{2}$,

gives $x^2 - \frac{x}{2}$,

and $-\frac{x}{2} + \frac{1}{4}$.

Sum of products is $x^2 - x + \frac{1}{4}$, as above.

43. What must be added to $x^2 - x$, to make the expression a perfect square?

44. What is the second root of $x^2 - x + \frac{1}{4}$?

45. What is the second power of $x - \frac{1}{3}$?

46. What must be added to $x^2 - \frac{2x}{3}$, to make the expression a perfect square?

47. What is the square root of $x^2 - \frac{2x}{3} + \frac{1}{9}$?

48. What is the second power of $x - \frac{2}{3}$?

49. What must be added to $x^2 - \frac{4x}{3}$, to complete the second power?

50. What is the second root of $x^2 - \frac{4x}{3} + \frac{4}{9}$?

51. What is the second power of $x - \frac{1}{5}$?

52. To what is $\sqrt{x^2 - \frac{4x}{5} + \frac{4}{25}}$ equal?

53. To what is $(x - \frac{1}{12})^2$ equal?

54. To what is $\sqrt{x^2 - \frac{x}{5} + \frac{1}{100}}$ equal?

SECTION XXXVII.

1. John and James had equal sums of money John lost two cents, and then the product of James's money multiplied by John's lacked but one cent of being a dollar. How much money had each?

Let x = the number of cents each had at first;

then $x - 2$ = the number John had left.

$x \times x - 2 \times x = x^2 - 2x$, the product of what each had left.

By the conditions of the question, $x^2 - 2x + 1 = 100$

Extracting the second root of each member,

$$x - 1 = 10.$$

Adding 1, $x = 11$ cents, each had.

2. One number is four less than another, and their product is forty-five. What are the numbers?

Let x = the greater;

then $x - 4$ = the less.

$x - 4 \times x = x^2 - 4x$, the product of the two.

By the conditions of the question, $x^2 - 4x = 45$

The square of each member must be completed, so that the second root of each member can be found. Twice the product of the two terms of the root, that is, $-4x$, must be divided by twice the square root of x^2, which is $2x$, and this will give -2 as the second term of the root; therefore, the square of -2, which is $+4$, must be connected with each member.

Adding 4, $x^2 - 4x + 4 = 49$.

Extracting second root, $x - 2 = 7$.

$x = 9$, the greater, and $x - 4 = 5$, the less

3. What must be added to each member of the equation $x^2 - 6x = 7$, to make each a perfect second power?

4. What is the second root of each member of the equation $x^2 - 6x + 9 = 16$? and what is the value of x?

5. What must be added to each member of the equation $x^2 - x = 12$, to make each a perfect square? What will the equation become? and what will be the value of x?

6. What must be added to the equation $4x^2 - 8x = 60$, that the square root of each member may be obtained? and what is the value of x?

7. In the equation $x^2 - \frac{x}{2} = 3$, what is the value of x?

8. In the equation $x^2 - \frac{x}{3} = \frac{2}{3}$, what is the value of x?

9. If each member of the equation $2x^2 - \frac{4x}{3} = 1\frac{1}{3}$ be divided by 2, what equation will represent the quotient?

10. If each member of the equation $3x^2 - 18x = 21$ be divided by 3, what will express the result? and what will be the value of x?

11. What must be added to $\frac{x^2}{4} - \frac{x}{4} = 3$, to make each member a perfect square? and what will be the value of x?

12. Ann had as many books as Jane; but Ann gave three of her books to Jane, and then Jane's number of books multiplied by Ann's number, less

twice the sum of their books, was twenty-three What number had each at first?

Let $x =$ the number each had; then $x - 3 =$ what Ann had left; and $x + 3 =$ what Jane then had. $\overline{x-3} \times \overline{x+3} = x^2 - 9$, the product of the two; but this product, less twice the sum of the books, that is, less $4x$, is equal to twenty-three books.

By conditions of the question, $x^2 - 9 - 4x = 23$.

Adding 9 to each member, $x^2 - 4x = 32$.

Completing the square, $x^2 - 4x + 4 = 36$.

Extracting the second root, $x - 2 = 6$.. $x = 8$, *Ans.*

13. If twice the square of some number be diminished by eight times the number, the remainder will be ten. What is the number?

14. A farmer sold six cows, and said, if the number he now had were multiplied by the number he had at first, he would have five more than one half of a hundred. How many cows had he at first?

15. If one half of a number be squared, and one half of the same number be subtracted from the square, the remainder will be two. What is the number?

16. Two men received the same wages for a week's work. But one spent two dollars, and then the square of the sum of what they both had left was one hundred and ninety-six dollars. What did each receive?

17. If from some number four be subtracted, and then one half of the remainder be multiplied by itself, the product will be only one. What is the number?

18. There is a fraction whose denominator is two

and if from the square of this fraction one fourth of the fraction be subtracted, the remainder will be one eighth. What is the numerator? and what the fraction?

Let $x =$ numerator, $\frac{x}{2} =$ the fraction.

19. One number is twice another. If six be subtracted from the greater, and then one sixth of the remainder be multiplied by itself, the product will be four. What are the numbers?

20. A man, being asked his age, said, that his age was one fourth of the square of his son's age, and that the difference of their ages was twenty-four years. What was the age of each?

21. A man changed a bank note, and spent one half of a dollar; he then found that the square of the money he had left was a quarter of a dollar more than twenty dollars. What was the value of the note?

22. The number of square feet in a square room is ninety-six more than the number of feet in the sum of its sides. What is the length of one side of the room?

23. If 5 be subtracted from each member of the equation $x^2 - 8x + 5 = 14$, what equation will express the remainder? What must now be added to each member, to make it a perfect second power? and what is the value of x?

24. What is the value of x in $x^2 - 4x + 7 = 103$?

25. If 3 be added to each member of the equation $x^2 - 2x - 3 = 45$, what will the equation become? What will the equation be when each member is made a perfect second power? and what will be the value of x?

SECTION XXXVIII.

1. When the value of x is 7, $x-4=3$. If each member be multiplied by itself, what will the equation be?

$\overline{x-4} \times \overline{x-4} = x^2-8x+16$, and $3 \times 3 = 9$, and the equation is $x^2-8x+16=9$.

2. When the value of x is 1, what will $x-4$ equal? It is evident that, if x represent 7 dollars, $x-4$ will equal 3 dollars, as above. If $x=4$ dollars, then $x-4$ will equal nothing; and if $x=1$, then $x-4$ must be represented by -3, or $x-4=-3$. For, if a man has only 1 dollar in his purse, and is called upon to pay a debt of 4 dollars, it is evident that he can pay but 1 dollar, which is the whole debt *less* 3 dollars; therefore -3 will represent the difference or deficiency.

3. When the value of x is 1, $x-4=-3$. If each member be multiplied by itself, what will the equation be?

$\overline{x-4}^2 = x^2-8x+16$, as above. Since a *minus* quantity multiplied by a *minus* quantity gives a *plus* product, -3×-3 will be $+9$, and the equation will be $x^2-8x+16=9$, as above. Thus, if the equation $x-4=3$, and $x-4=-3$, each be squared, the *same* equation will be produced, namely, $x^2-8x+16=9$.

4. If the square root of $x^2-8x+16=9$ be extracted, what will the equation be?

$\sqrt{x^2-8x+16}=x-4$; $\sqrt{9}=-3$, or $+3$, since either multiplied by itself will produce $+9$. Therefore, $x-4$, the square root of the first member, will equal either *plus* 3 or *minus* 3, the root of the second member; and the equation may be expressed thus; $x-4=\pm 3$.

5. If $x-4=\pm 3$, what is the value of x?

Adding 4 to each member, $x=4\pm 3$, and the value of x is 4 *plus* or *minus* 3. If the root is $+3$, or *positive*, then $x=4+3$, or 7; and $x-4=3$ will be $7-4=3$, as in Example 1st. If the root be -3, or *negative*, then $x=4-3$, or 1, as in Example 3d. The value of x is either 7 or 1. This may be verified by substituting each of these values for x, in the original equation, $x^2-8x+16=9$. Putting 7 for x, gives $49-56+16=9$; and putting 1 for x, $1-8+16=9$. Here each value of x accords with the algebraic expression.

Remark. — Hence in an equation of the second degree, the unknown quantity will have two different values, either of which, when substituted, will satisfy the algebraic expression; while only one of them will generally satisfy the conditions of the question. How to determine whether the root be positive or negative, and what is the true value of the unknown quantity, may be seen in the solution of the following problem.

6. A man paid a debt of four dollars, and then found that the square of the money left in his purse was nine dollars. How many dollars had he at first?

Let $x=$ his money; then $x-4=$ what he had left after paying the debt; then the square of $x-4$ must equal 9 dollars. $\overline{x-4}^2$ is $x^2-8x+16$.

By the conditions of the question,

$$x^2 - 8x + 16 = 9.$$

Extracting square root, $x - 4 = 3$, and $x = 4 \pm 3$.

If the root of 9 be $+3$, $x = 4 + 3 = 7$: therefore he had 7 dollars; and, after paying the debt of 4 dollars, he had $7 - 4 = 3$ dollars left, the square of which is 9 dollars. This agrees with the conditions of the question, and is the true value of x.

If the root of 9 be -3, $x = 4 - 3$, or 1; therefore, he had 1 dollar at first, and if he had but 1 dollar, he could not pay the required debt of 4 dollars, and have the required sum left. This last value will not satisfy the conditions of the question, and cannot be the true value.

7. A boy bought some oranges. If he had bought two less at the same rate, the number of oranges would have been equal to the price of one orange, and they would have cost him thirty-six cents. How many did he buy? and what did each cost?

8. If twice some number be subtracted from its square, the remainder will be thirty-five. What is the number?

9. Boston and Providence are forty miles apart. A man starts from Boston, and, after travelling a number of miles, finds that, if twice the distance he has travelled be subtracted from the square of that distance, one half of the remainder will be the whole distance from Boston to Providence. How far had he travelled?

10. If from four times the square of a fraction, one third of the fraction be subtracted, the remainder will be $\frac{1}{3}$. What is the fraction? Let $x =$ the fraction

11. A and B received the same sum of money for a week's wages. A spent two dollars, and B four dollars; then the product of A's money multiplied by B's was forty-eight dollars. How much money did each receive?

12. If to the square of one half of some number one third of the same number be added, the sum will be eleven. What is the number?

13. The number of cents that A paid for a melon was equal to half the number of melons that he bought. B bought four more than A, at the same price, and they cost him four cents less than a dollar. What was the price of a melon? and how many melons did each buy?

14. If half of some number be added to the square of half of the same number, the sum will be $3\frac{3}{4}$. What is the number?

15. A travelled twice as far as B. If A had travelled four miles farther, and if that distance were multiplied by the number of miles B travelled, the product would be one hundred and twenty-six miles. How many miles did each travel?

16. If from the square of some number twice the number be subtracted, the remainder will be seven more than four times the number. What is the number?

17. What is the value of x in the equation $3x^2 - 6x - 18 = 12x + 30$?

SECTION XXXIX.

1 George and his brother have $10. If George's money be multiplied by his brother's, the product will oe $24. How much money has each?

Let $x =$ George's; then $10 - x =$ his brother's, and $\overline{10 - x} \times x =$ their product. By the conditions of the question, $24 = 10x - x^2$; adding x^2, and subtracting 24, $x^2 = 10x - 24$; subtracting $10x$, $x^2 - 10x = -24$.

Hence it is evident that, if the signs before all the terms of each member be changed, the equation will still be preserved. For, since x^2 is 24 less than $10x$, if we try to take $10x$ from x^2, 24 will be wanting, as expressed by -24.

Completing the square, $x^2 - 10x + 25 = 25 - 24 = 1$.

Extracting square root, $x - 5 = \pm 1$, and $x = 5 \pm 1$.

If the root of 1 is *plus*, or *positive*, $x = 6$, or George's, and $10 - x = 4$, or his brother's. But if the root is *minus*, or *negative*, $x = 4$, or George's, and $10 - x = 6$, or his brother's. Either will answer the conditions of the question, as it does not specify which had the most money.

2. Divide 12 into two such parts, that the square of the less will be equal to twice the greater. If x represents the less, what equation will be formed? What must be added to each member, that only the terms containing the unknown quantity may constitute one member? What are the parts?

3. The united ages of two boys are 15 years, and the square of the age of the younger boy is 5 years more than twice the age of the elder. How old is each?

4. Find two numbers whose sum shall be 20, and the square of one third of the greater shall be double the less. What are the numbers?

5 The sum of the distances that Peter and John walked is 8 miles, and the product of the distances is 12 miles. What distance did each walk?

6. The sum of two numbers is 7, and if the greater be multiplied by the less, the product will be 3 more than their sum. What are the numbers?

7. A has more money than B, and they both together have $14. The square of A's money is $24 less than 14 times his money. How many dollars has each?

8. What number is that, to the square of which if 21 be added, the sum will be 10 times the number?

9. A man had $10; he spent a part of it, and the square of what he spent was 9 times what he had left. How many dollars did he spend?

10. What number is that, to the square of which if you add 60, and then divide the sum by 16, the quotient will be the number itself?

11. A and B, together, build 7 rods of wall, and each, by agreement, receives as many dollars per rod as the number of rods he builds. A received $2 less than double what B received. How many rods did each build? and how many dollars did each receive?

12. George and Charles, together, bought 10 books, and each paid as many cents for one of his books as was equal to the number of books he bought. George spent 2 cents less than half the money which Charles spent. How many books did each buy? and how much money did each spend?

SECTION XI.

1. The sum of the ages of John and William is 8 years, and the product of their ages is 15. How old is each?

Let $x =$ John's, $y =$ William's; then $xy =$ their product.

(1.) By one condition of the question, . . $xy = 15$

(2.) By another condition, ... $x + y = 8$

(3.) Subtracting y from 2d, ... $x = 8 - y$

(4.) Dividing 1st by x, ... $y = \frac{15}{x}$

(5.) Substituting this value of y, in 3d, $x = 8 - \frac{15}{x}$.

(6.) Multiplying 5th by x, ... $x^2 = 8x - 15$.

(7.) Subtracting $8x$ from 6th, . . . $x^2 - 8x = -15$

(8.) Completing the square of 7th, $x^2 - 8x + 16 = 16 - 15$, or 1.

(9.) Extracting 2d root of 8th, ... $x - 4 = \pm 1$

$x = 4 \pm 1$; therefore, $x =$ either 5 or 3.

Putting 5 for x in 4th, $y = 3$, and 3 for x, $y = 5$.

As the question did not specify the elder, either value will answer its conditions; therefore, John is 3 years and William 5; or John 5, and William 3.

2. The sum of two numbers is 10, and their product is 21. What are the numbers?

3. John is 2 years older than William, and the product of their ages is 15. How old is each?

4. The sum of two numbers is 20, and their product is 96. What are the numbers?

5 The sum of two numbers is 6, and the sum of their squares is 20. What are the numbers?

6. The sum of two numbers is 6, and the difference of their squares is 12. What are the numbers?

7. The sum of two numbers is 6, and their product is 8. What are the numbers?

8. The difference of two numbers is 1, and the difference of their squares is 9. What are the numbers?

9. The greater of two numbers divided by the less, is equal to the less, and the difference of their squares is 72. What are the numbers?

10. A travelled 5 miles less than B, and the product of the distances both travelled is 84 miles. How many miles did each travel?

11. What is that fraction which will be equal to $\frac{1}{2}$, if 2 be added to its numerator, and if the numerator be taken from the denominator, the difference will be 7?

Let $x =$ numerator, $y =$ denominator, $\frac{x}{y} =$ the fraction.

12. There are two numbers, such that, if the greater be divided by the less, the quotient will be 3, and their difference is 4. What are the numbers?

13. There are two numbers whose sum is 5, and whose product is 6. What are the numbers?

14. There are two numbers, such that, if $\frac{1}{4}$ of the greater be added to $\frac{1}{2}$ of the less, the sum will be the less number, and their product is 32. What are the numbers?

15. If 1 be added to the denominator of a fraction, the fraction will be $\frac{1}{2}$, and the produc of the numer

ator multiplied by the denominator is 6. What is the fraction?

16. George is 1 year older than Anna, and the difference between the squares of their ages is 19. How old is each?

17. The product of two numbers is 12, and their difference is 1. What are the numbers?

18. If the greater of two numbers be multiplied by the less, the product will be 10, and the difference between the two numbers is 3. What are the numbers?

19. A boy bought an orange and 3 lemons for 11 cents, and the price of a lemon multiplied by the price of an orange was 10 cents. What was the price of one of each?

SECTION XLI.

1. If x be multiplied by x, the product is x^2, the second power or square of x. If x^2 be multiplied by x, the product will be x^3, that is, the third power or cube of x; and the third root or cube root of x^3 must be x.

2. What is the product of $x \times x \times x$?

3. What is the product of $3 \times 3 \times 3$? or what is the cube of 3?

4. What is the third or cube root of 27?

5. What is the cube or third power of 2?

6. What is the cube or third power of 4?

7. What is the cube root of 8?

8. What is the third root of 64?

9. What is the cube of $2x$?

10. What is the cube of $4x$?

11. What is the third power of $5x$?

12. What is the third root of $64x^3$?

13 What is the cube root of $8x^3$?

14 What is the cube root of $125x^3$?

15 If x^2 be divided by x, the quotient will be x. If x^3 be divided by x, what will the quotient be?

16. Divide x^3 by x^2, what will the quotient be?

17. Divide $64x^2$ by $4x$, what will be the quotient?

18. Divide $125x^3$ by $25x^2$, what will be the quotient?

19. Extract the cube root of $8x^3$?

20. Extract the third root of $27x^3$?

21. The product of $\frac{x}{2} \times \frac{x}{2}$ is $\frac{x^2}{4}$, and $\frac{x^2}{4} \times \frac{x}{2}$ is $\frac{x^3}{8}$. What is the cube root of $\frac{x^3}{8}$?

22. What is the product of $\frac{x}{3} \times \frac{x}{3} \times \frac{x}{3}$?

23. What is the third power of $\frac{x}{2}$?

24. What is the third root of $\frac{x^3}{8}$?

25. What is the cube root of $\frac{8x^3}{64}$?

26. What is the third power of $\frac{x}{4}$?

27. What is the cube of $\frac{3x}{4}$?

28. What is the cube root of $\frac{x^3}{64}$? of $\frac{27x^3}{64}$?

29. What is the third power of $\frac{x}{5}$? of $\frac{2x}{5}$? of $\frac{3x}{5}$?

30. What is the cube root of $\frac{x^3}{125}$? of $\frac{27x^3}{125}$?

31. Divide $\frac{27x^3}{125}$ by $\frac{9x^2}{25}$, what will the quotient be?

32. If $x=2$, to what will the cube of x be equal?

33. If $x=3$, what will x^3 equal?

34. If $x^3=8$, what will x equal?

35. What is the cube root of the equation $x^3=27$?

36. If $x=4$ be raised to the third power, what will the equation be?

37. What is the cube root of $x^3=125$?

38. In the equation $x^3=64$, what will be the value of x?

39. What is the third power of $\frac{x}{2}=3$?

40. What is the cube of the equation $\frac{2x}{3}=4$?

41. What is the cube root of the equation $\frac{x^3}{27}=27$?

42. In the equation $\frac{8x^3}{27}=64$, what is the value of x?

43. In the equation $\frac{27x^3}{64}=64$, what is the value of x?

44. In the equation $8x^3=64$, what is the value of x?

45. In the equation $27x^3=27$, what is the value of x?

46. If the equation $x^3=16x$ be divided by x, what will be the result? What will be the value of x?

47. If x^2 be multiplied by x^2, the product is x^4. What will be the product of $2x^2$ multiplied by $2x^2$?

48. What is the fourth power of x, or $x \times x \times x \times x$?

49. What is the square root of x^4? of $16x^4$?

50. What is the fourth root of x^4? of $16x^4$?

51. What is the fourth power of $3x$? of $2x$? $\frac{x}{2}$? of $\frac{x}{3}$?

52. What is the fourth root of $81x^4$? of $\frac{x^4}{16}$?

SECTION XLII.

1. A MAN, being asked the age of his son, said, "If the square of his age be multiplied by his age, the product will be 81 times his age." What was the son's age?

Let $x =$ his age;

then $x \times x = x^2$, the square of his age.

By the conditions of the question, $x^2 \times x = 81x$, or

$$x^3 = 81x.$$

Dividing by x, $x^2 = 81$.

Extracting square root, $x = 9$, *Ans.*

2. If $x^3 = 81x$, what does x^2 equal? What does x equal?

3. The cube of a number is 27. What is the number?

Let $x =$ the number; then $x^3 = 27$.

Extracting the cube root of each member, $x = 3$.

4. If x be multiplied twice by x, it will be expressed thus; $x \times x \times x = x^3$. What will express the product of $2x$ multiplied by itself twice?

5. If a boy's money be taken from the cube of his money, the remainder will be 15 times his money. How many dollars has he?

6. The cube of a number is 4 times the number What is the number?

7. The cube of a number is 16 times the square of the same number. What is the number?

8. The cube of a number is 64. What is the number?

9. What must be the side of a cubical box to contain 125 cubic feet?

10. The cube of one half of a number is twice the number itself. What is the number?

11. If the second power of a number be multiplied by $\frac{1}{4}$ of the number, the product will be 16. What is the number?

12. A's age is the square of B's, and C's is the product of A's multiplied by B's, and the sum of their ages is 21 times B's age. What is the age of each?

13. If from the third power of a number 4 times the second power of the same number be subtracted, the remainder will be 4 times the square of the number. What is the number?

14. If from the cube of a number the square be subtracted, the remainder will be 6 times the number. What is the number?

15. If from the cube of some number 60 be subtracted, only 4 will remain. What is the number?

16. What must be the side of a cubical box containing 216 cubic feet?

17 A man said, if 10 times his money were taken

from the cube of his money, the remainder would be 9 times the square of his money. How many dollars had he?

18. The cube of a number, less 25, is 100. What is the number?

19. A man, being asked the age of his son, said, "If 3 times the square of his age be taken from the fourth power of his age, the remainder will be 6 times the square of his age." How old was his son?

20. A's money is the cube of B's, and if 20 times B's be taken from A's, the remainder will equal the square of B's. How many dollars has each?

SECTION XLIII.

1. Since $\frac{4}{2} = 2$, and $\frac{6}{3} = 2$, therefore $\frac{4}{2} = \frac{6}{3}$. Here 2 has the same *relation* or *ratio* to 4, that 3 has to 6. This relation may be expressed thus;

$$2 : 4 = 3 : 6;$$

that is, the ratio of 2 to 4 equals, or is the same as, the ratio of 3 to 6.

2. In the above *equation of ratios*, 4 is the same part of 2 that 6 is of 3, and 2 is the same part of 4 that 3 is of 6.

3. From the above *proportion*, or *equality* of *ratios*, it is apparent that the *product* of the *extremes* that is, 2×6, is equal to the *product* of the *means*, that is, 4×3; or $2 \times 6 = 4 \times 3$, since these products are the same.

4. To what number has 3 the same ratio or relation that 2 has to 4?

Let $x =$ the number; then $2 : 4 = 3 : x$. Multiplying extremes and means, as in 3d, $2x = 12$, and $x = 6$, *Ans.*

5. What number has the same ratio to 6 as 2 to 4?

$x =$ number; then $2 : 4 = x : 6$; $4 \times x = 2 \times 6$, or $4x = 12$; $x = 3$, *Ans.*

6. To what number has 2 the same ratio as 3 to 6?

$x =$ number; then $2 : x = 3 : 6$; $3 \times x = 2 \times 6$, or $3x = 12$; $x = 4$, *Ans.*

7. What number has the same ratio to 4 as 3 to 6?

$x =$ number; then $x : 4 = 3 : 6$; $6 \times x = 4 \times 3$, or $6x = 12$; $x = 2$, *Ans.*

8. Thus, if any three terms in an equality of ratios be known, the other may be found; that is, dividing the product of the means by one extreme, gives the other extreme, and dividing the product of the extremes by either one of the means, gives the other. Hence the "Rule of Three," or "Proportion."

9. A man gave $8 for 4 sheep. What will 5 sheep cost at the same rate?

Let $x =$ cost of 5 sheep; then $4 : 8 = 5 : x$. By the 3d, $4 \times x = 5 \times 8$, or $4x = 40$. $x = 10$. *Ans.* $10.

10. If 2 oranges cost 8 cents, what will 7 oranges cost?

11. If 4 writing-books cost 24 cents, what will 3 cost?

12. If 2 cows cost $40, what will 5 cows cost?

13. Two numbers are to each other as 3 to 4, and their product is 48. What are the numbers?

Let $x =$ less, and $y =$ greater; then $x : y = 3 : 4$, and $xy = 48$. By 3d, $4 \times x = 3 \times y$, or $4x = 3y$. $x = \frac{3y}{4}$. Put $\frac{3y}{4}$ for x, $\frac{3y}{4} \times y = 48$, or $\frac{3y^2}{4} = 48$. $\frac{y^2}{4} = 16$. $y^2 = 64$. $y = 8$. Put 8 for y, $x = \frac{24}{4} = 6$. *Ans.* 6 and 8.

14. If x is to y as 2 to 5, what part of y is x?

$x : y = 2 : 5$. By 3d, $5x = 2y$. $x = \frac{2y}{5}$, or $\frac{2}{5}$ of y, *Ans.*

15. If one number is to another as 3 is to 7, and if x represents the greater, what part of x will represent the less?

Let $y =$ the less; then $y : x = 3 : 7$. By 3d, $7y = 3x$. $y = \frac{3x}{7}$, and $\frac{3x}{7}$ will represent the less number.

16. Two numbers are to each other as 3 to 4, and their difference is 3. What are the numbers?

17. Two numbers are to each other as 1 to 3, and the square of their sum is 64. What are the numbers?

Let $y =$ smaller, and $x =$ larger; then $y : x = 1 : 3$. By 3d, $3y = x$. $y = \frac{x}{3}$; then $x =$ larger, and $\frac{x}{3} =$ smaller. $x + \frac{x}{3} = \frac{4x}{3}$, their sum. By the question, $\frac{16x^2}{9} = 64$. $\frac{4x}{3} = 8$. $x = 6$. $\frac{x}{3} = 2$.

Ans. 2 and 6.

18. Two numbers are to each other as 2 to 3, and

the difference of their squares is 20. What are the numbers?

19. Two numbers are to each other as 4 to 7. If the less be subtracted from the greater, the remainder will be 6. What are the numbers?

20. John's money was to William's as 1 to 3. William spent 10 cents, and then John's was to William's as 1 to 2. How much money had each at first?

21. The difference of two numbers is to their sum as 3 to 13, and their product is 40. What are the numbers?

22. A's number of horses multiplied by B's would be 15. A sold one horse to B, and then A's horses were to B's as 1 to 3. How many horses had each at first?

23. The product of two numbers is 8, and their squares are to each other as 1 to 4. What are the numbers?

MISCELLANEOUS QUESTIONS.

1. Anna is 3 times as old as Charles, and the sum of their ages is 12 years. What is the age of each?

2. What number must be added to 5 times itself, that the product may be 54?

3. If a number be added to $\frac{1}{2}$ of itself, the sum will be 27. What is the number?

4. If a number be increased by $\frac{2}{5}$ of itself, the sum will be 21. What is the number?

5. One number is 6 more than another, and their sum is 28. What are the numbers?

6. One number is 7 less than another, and their sum is 23. What are the numbers?

7. The sum of 2 numbers is 33, and their difference is 9. What are the numbers?

8. Daniel lost $\frac{2}{5}$ of his money, and had 12 cents left. How many cents had he at first?

9. Levi says, the difference between $\frac{2}{5}$ and $\frac{3}{4}$ of his age is 7 years. How old is he?

10. A man paid away $\frac{2}{3}$ of his money, and lost $4. He still had a dollar left. How many had he at first?

11. John says, "I have $\frac{3}{5}$ of my books left; and if you give me 10 more, I shall have my original number complete." How many had he at first?

12. A farmer says, "If I had as many more sheep as I now have, $\frac{1}{2}$ as many more, and $\frac{3}{5}$ as many more, I should still lack 7 of having a hundred." How many has he?

13. Frederic says, if you will give him 5 more

apples, he can divide what he will then have among his 3 companions, and they will get 7 apples apiece. How many apples has he?

14. In an orchard of 120 trees there are twice as many pear-trees as peach-trees, and 3 times as many apple-trees as there are of both the other kinds. How many trees are there of each kind?

15. A man paid $\frac{2}{5}$ of his money to one person, and $\frac{1}{4}$ of it to another, and still had $28 left. How many dollars had he at first?

16. A boy gave away $\frac{1}{4}$ of his money, and spent $\frac{1}{3}$ of it. He then had 20 cents left. How much money had he at first?

17. Says John to Samuel, "My age is now only $\frac{1}{2}$ of yours; but if we live 4 years longer, mine will be $\frac{2}{3}$ of yours." What is the age of each?

18. A man on horseback travelled a certain distance in 15 hours. A locomotive, going at the rate of 20 miles an hour, travelled the same distance in 3 hours. How many miles an hour did the man on horseback travel?

19. A traveller started from Boston for Albany 9 hours before the cars, and the train, going at the rate of 18 miles an hour, overtook him in 4 hours. How many miles did he travel in an hour?

20. A says to B, "Give me $\frac{1}{3}$ of your money, and I can spend $2, and still have remaining double what you would have left." "How is that?" says B; "for I have $\frac{3}{4}$ as many as you now." How much money has each?

21. Two men started, at 6 o'clock in the morning, one from Philadelphia, and the other from New York,

90 miles apart, to meet each other. A travelled 4, and B 5 miles an hour. At what time did they meet? and how far did each travel?

22. Divide 17 into 2 such parts, that $\frac{3}{4}$ of the one shall be equal to $\frac{2}{3}$ of the other. What are the parts?

23. A man, travelling 5 miles an hour has 10 hours the start of a train of cars, going at the rate of 5 miles to the man's 1. In how many hours will the train overtake the man? and how far must it go to do so?

24. Five years ago, Kate was twice as old as Abby. Now Abby's age is to Kate's as 2 to 3. What are their ages?

25. A receives $\frac{2}{3}$ as much money as B. After A had spent $2, B had double what A had left. How many dollars had each at first?

26. A revenue cutter, in pursuit of a merchant ship, sails 3 miles to the ship's 2; but the ship goes at the rate of 6 miles an hour, and has 3 hours the start of the cutter. In how many hours will the ship be overtaken? and how many miles must the cutter sail to do it?

27. If 5 be added to 5 times a number, $\frac{1}{7}$ of the sum will be 1 less than the number. What is the number?

28. A can plant $\frac{1}{5}$ of a field in a day, and B can plant $\frac{1}{6}$ of it in the same time. If they work together, how long will it take them to plant it?

29. Says Samuel to William, "Give me 1 of your apples, and my number will be double of yours. William replies, "Give me 1 of yours, and we shall each have the same number." How many has each?

30 A boy wished to buy a certain number of pencils, at 4 cents apiece, but lacked 3 cents of being able to pay for them; so he bought the same number, at 3 cents apiece, and had just money enough left to buy one more at the latter price. How much money had he? and how many pencils did he buy?

31. What o'clock is it when the minute-hand and hour-hand are together for the fourth time since 12 o'clock?

32. A boy has some money in each hand, and $4 in his purse. When he takes the purse in his right hand, the money in that hand is double the money in his left hand; but when the purse is in his left hand, the money in the left is $2 more than there is in the right hand. How many dollars has he in each hand?

33. John bought 5 peaches and 3 pears for 21 cents. Andrew, with only $\frac{1}{3}$ as much money, bought, at the same rate, 2 pears and 1 peach. How much did they pay for 1 of each kind of fruit?

34. Three times Eliza's age added to twice Abby's age is 27 years. If three times Abby's age be taken from twice Eliza's, the difference will be 5 years. What is the age of each?

35. A steamboat, in pursuit of a ship, sails 3 miles while the ship sails 2; but the ship started 5 hours before the steamboat, and averages 8 miles an hour. How many miles must the steamboat go to overtake the ship? and how many hours will it take to do it?

36. A farmer has his cows in 2 pastures, and one pasture has in it $\frac{2}{3}$ as many as the other. He took 1 cow out of the pasture containing the less number

and put her in the other; and then the latter contained double the number in the former. How many cows were in each pasture at first?

37. The quotient of one number divided by another is 3, and their difference is 4. What are the numbers?

38. The length of a room is 9 feet more than the breadth, and the number of square feet in it equals 10 times the length of the room. What is the length of the room?

39. The difference between two numbers is 3, and their product is 28. What are the numbers?

40. A man bought 3 calves and 4 sheep for $27. He afterwards, at the same rate at which he purchased, returned 2 calves and 1 sheep to the seller, and received back $13. What was the price of 1 of each?

41. If $\frac{2}{3}$ of a number be multiplied by $\frac{3}{4}$ of the same number, the product will be 72. What is the number?

42. A farmer said, if $\frac{1}{2}$ his number of cows were multiplied by $\frac{1}{5}$ of the number, the product would be his number of cows. How many had he?

43. If 1 be added to the quotient of 10 divided by some number, the sum will be 3 times that number? What is the number?

44. If $\frac{1}{4}$ of a number be multiplied by $\frac{1}{2}$ of the same number, and from the product $\frac{3}{4}$ of the number be taken, the remainder will be 2. What is the number?

45. The sum of two squares is 100, and their difference is 28. What are their square roots?

46. Divide 39 into two such parts, that the greater

divided by the square of the less, shall be equal to 3 What are the numbers?

47. Matilda had as much money as Catharine, but Catharine gave $3 to Matilda, and then Matilda's money, multiplied by Catharine's, was $40. How many dollars had each?

48. One number is to another as 1 is to 2, and their product, less the smaller number, is 6. What are the numbers?

49. If John's money be taken from 3 times Henry's, and $\frac{1}{3}$ of the remainder be added to $\frac{1}{4}$ of the difference between Henry's and twice John's, the sum will be $14; but half of John's money is $2 more than $\frac{1}{4}$ of Henry's. How many dollars has each?

50. The difference between the numerator and denominator of a proper fraction is 6; and if 2 be taken from the numerator, and added to the denominator, the fraction will be $\frac{1}{3}$. What is the fraction?

51. What o'clock is it when the square of the time past from midnight is equal to the remaining time to noon?

52. If the denominator of a fraction be divided by the numerator, the quotient will be 4; and if the numerator be multiplied by the denominator, the product will be 4. What is the fraction?

53. One number is the square of another, and if the less be increased by 2, and the sum multiplied by the greater, the product will be 24 times the smaller. What are the numbers?

54. A man started from Boston to go to Hartford, a distance of 100 miles. After travelling a short time, he found, if $2\frac{1}{2}$ times the distance he had trav

elled were taken from the square of half that distance, the remainder would be $\frac{2}{3}$ of the whole distance to Hartford added to half the distance he had travelled. How far had he travelled?

55. The difference of two numbers, multiplied by the less, is twice the less, and twice the greater added to the less is 16. What are the numbers?

56. A had 2 dollars to B's 3; and after counting their money, they found that, if 2 dollars were taken from half of A's money, and the remainder were multiplied by $\frac{2}{3}$ of B's, the product would be double B's money. How many dollars had each?

57. A room contains 120 square feet, and the difference between the length and the width is 2 feet. How long and how wide is the room?

58 The product of two numbers is 50, and their quotient is 2. What are the numbers?

59. The length of a fence is 10 times its height, and the number of square feet in the fence is equal to twice the cube of the height. How long and how high is the fence?

60. The product of Sarah's money, multiplied by $\frac{1}{4}$ of Eliza's, is equal to Sarah's, and the square of Eliza's is $20 less than the square of Sarah's. How many dollars has each?

61. On asking the number of cannon balls in a certain pile at the Navy Yard, an officer replied, "If $1\frac{1}{6}$ of the number of balls be multiplied by $\frac{1}{5}$ of the number, and from the product $\frac{3}{5}$ of the number be taken, the remainder will be 20 balls." How many balls were there in the pile?

62. Two men left Philadelphia for Baltimore, a dis-

tance of 100 miles, at 5 o'clock, A. M. A travelled 2 miles an hour faster than B, and B was $2\frac{1}{2}$ hours longer on the way. How fast did each travel? and at what o'clock did each reach Baltimore?

63. Divide 14 into 2 such parts, that if the product of the parts be divided by the second power of the smaller part, the quotient will be to the greater part as 1 is to 4. What are the parts?

64. The difference between the length and breadth of a room is 5 feet. If 9 feet be taken from $\frac{3}{5}$ of the number of square feet in the room, the remainder will be the number of square feet in a square room, whose side is 1 foot less than the breadth of the room whose dimensions are required. How long and how wide is the room?

65. There is a bridge 100 rods long, and the square of one fourth of the distance from the north end of the bridge to the middle of the draw, less once that distance, is equal to the distance from the middle of the draw to the other end of the bridge. How far is the middle of the draw from each end of the bridge?

66. If A's money, which is $12, be divided by B's money less $2, the quotient will be $1 less than B's money. How many dollars has B?

67. Peter is 4 years older than John, and half the product of their ages, added to the sum of their ages is equal to the sum of Peter's age, added to the square of John's age. What are their ages?

68. What o'clock is it when the time past from noon to midnight, taken from the square of the time past, is equal to the time from noon to midnight?

Recommendations and Notices

OF

TOWER'S INTELLECTUAL ALGEBRA.

The subscribers, Principals in the Department of Mathematics in the Public Schools of Boston, have examined D. B. Tower's "*Intellectual Algebra*," and are well pleased with the Work. They believe that the careful and minute analysis of questions in it is calculated to train the mind of the pupil to correct habits of investigation, and they cordially recommend it to the consideration of those interested in education.

PETER MACKINTOSH, JR.
LEVI CONANT,
JOSIAH FAIRBANK,
REUBEN SWAN, JR.
LORING LATHROP,
JOSEPH HALE,
JONATHAN BATTLES, JR.
JAMES ROBINSON,
AARON D. CAPEN,
NATHAN MERRILL,
JOHN A. HARRIS,
CHARLES KIMBALL,
WILLIAM A. SHEPHARD
BENJAMIN DREW, JR.

June 28th, 1845.

BOSTON, JUNE 30th, 1845.

We have examined the "*Intellectual Algebra*," by D. B. Tower, and we are glad to find that the hitherto perplexing science of Algebra is so simplified and so clearly illustrated, as to render it easily attainable by the younger classes of children.

Mr. Tower has the merit of originality in his conception of an "*Intellectual Algebra*." The value of this work is much enhanced, not merely from the fact that the author ranks high as a Mathematician; but in an especial manner, since he has been a successful Teacher in this department, and is thoroughly versed in the best modes of presenting the subject to the minds of his pupils in the various forms of practical instruction.

The work is systematic in its arrangement; it contains all that will be useful in Common Schools, and is just what is wanted to make a *thinking* pupil. We can, therefore, commend it to the notice and patronage of Teachers, Parents, and School Committees; believing that where it is used the pupils will acquire not only a competent knowledge of Algebra, but, at the same time, they will be making as much progress in Arithmetic, as they could, if required to give their *exclusive* attention to the best text-books now used in Oral Arithmetic.

CORNELIUS WALKER,
SAMUEL BARRETT,
ABNER FORBES,
CHARLES B. SHERMAN,
THOMAS BAKER,
JOSHUA BATES, JR.,
GEORGE B. HYDE,
RICHARD G. PARKER,
W. J. ADAMS,
FREDERICK CRAFTS,
ALBERT BOWKER,
JOSIAH A. STEARNS,
ISAAC F. SHEPARD,
Grammar Masters.

CHARLESTOWN, JULY 11, 1845.

Dear Sir,—I have the pleasure to inform you that after a careful examination on the part of our Board of Trustees, of your "*Intellectual Algebra*," it was unanimously voted to introduce it into our Grammar Schools. Some of our Teachers have thoroughly examined the book, and speak in high terms of its merits.

Respectfully yours, JONATHAN BROWN, JR.,
Secretary

To D. B. TOWER, Esq.

Mr. PIERCE, the experienced Principal of the Normal School, West Newton, June 26th, writes, "I am so well pleased with it (the Algebra), that I propose to introduce it into the Model School next Term."

CHELSEA, JULY 9, 1845.

Mr. Tower,—Dear Sir: I have examined your "*Intellectual Algebra*," and I should be much gratified at its introduction into the School under my charge. I find the mental exercises in the Arithmetic we use altogether inadequate, and am confident that the introduction of your work, at this stage of the scholar's progress, will enable him to understand the science of Arithmetic much better and more easily than he can now do.

Respectfully, QUINCY ADAMS.

CHARLESTOWN, JULY 8, 1845.

Mr. Tower,—Dear Sir: Your work on "*Intellectual Algebra*," we have examined with much interest, and a high degree of pleasure The idea of the work is excellent, and the arrangement, we think is good.

It is the first book of the kind that we have seen, and it appears to be well calculated to supply *a deficiency* in the class of books for the intellectual training of the youthful mind. A more interesting, useful, and important work could hardly have been devised, and it cannot fail, we think, to meet the approbation of Teachers and friends of education.

Very respectfully, P. H. SWEETSER,
Principal of Grammar Department of Harvard School.
DANIEL H. FORBES,
Principal of Grammar Department of Warren School
A. WALKER,
Principal of Grammar Department of Winthrop School

CHARLESTOWN, JULY 19, 1845.

We have examined, carefully and with much satisfaction, Tower's '*Intellectual Algebra*," which bears the same relation to the Algebraic text-books in common use, as that sustained by "Colburn's First Lessons" to previous treatises upon Arithmetic—and we think that every one, who has made use of that excellent work, cannot fail to regard this as the highest commendation. We are highly gratified

to learn that the Trustees have introduced the work into the Schools under our care.

BENJAMIN F. TWEED,
Principal of Bunker Hill School.
JOSEPH T. SWAN,
Principal of Mathematical Department of Warren School.
STACY BAXTER,
Principal of Mathematical Department of Winthrop School.

From Professor Forbes, Civil Engineer, formerly Principal of the High School in Lowell.

LOWELL, JULY 21, 1845.

Dear Sir—I have examined your "*Intellectual Algebra*" with interest; and I believe it will be found highly useful in giving to the young habits of thinking attentively, and of reasoning with precision—two of the most desirable results of education. Your book is the best of its kind that I have seen.

Very respectfully Yours, FRANKLIN FORBES.

DAVID B. TOWER, Esq.

SALEM, JULY 12, 1845.

D. B. Tower, Esq.—Dear Sir: I have examined with much attention your "*Intellectual Algebra.*" I think the plan of the work is excellent; and so far as I have examined, the filling up is equally good. I suspect you have done for Algebra a service not very unlike what Colburn did for Arithmetic, when he published his "*First Lessons.*" I have requested our School Committee to allow me to put it into the hands of my Junior Class, as a preparatory study.

Yours, very respectfully, RUFUS PUTNAM,
Principal of the Bowditch English High School, Salem, Mass.

Boston Daily Journal.

The plan of this work is altogether new—it contemplates the improvement in the mode of teaching Algebra, that Colburn introduced into Arithmetic some twenty years ago, viz.—by *oral* exercises, in which all the operations are limited to such small numbers as not to embarrass the reasoning powers, but on the *inductive plan*, to lead the pupil, understandingly, step by step, to higher mental efforts * * * * We think its merits will be found to entitle it to admission into our schools as a valuable aid to the Teachers in giving instruction in Algebra to our youthful readers.

Mass. Temperance Standard, Aug. 1, 1845.

We have looked over this work with much interest. To most persons, the idea of the study of Algebra, is that of a hard, dry, useless task; and formerly this idea was in the main correct. Some of the early treatises on this subject seem to have been intended to convey the little information they contained, in as blind a method as possible. But Warren Colburn, by his excellent treatise, made the translation from the study of Arithmetic to that of Algebra, easy

and delightful. Not content with this advance, Mr. Tower has now prepared a treatise, which is designed to hold the same position in reference to Algebra that Mr. Colburn's "*Intellectual Arithmetic*" does to Arithmetic—that is, to make it one of the most elementary studies in common schools. The idea seems to us a good one. There is nothing in the nature of Algebra to render it a difficult study. If any one doubts this statement, let him read over Mr. Tower's book, and he will be sceptical no longer. But what is of still higher importance, the child by these steps, which seem so pleasant and simple, is learning the greatest of all arts—that of reasoning. In this age of loose reasoners, every man who does anything to direct the minds of the young to habits of closer investigation and analysis, does a service to the community which cannot easily be over-rated. In this respect it gives us great pleasure to recommend the little treatise of Mr. Tower.

Boston Messenger, July 31, 1845.

"*Intellectual Algebra;* or, Oral Exercises in Algebra, for Common Schools—in which all the operations are limited to such small numbers as not to embarrass the reasoning powers, but, on the inductive plan, to lead the pupil understandingly, step by step, to higher mental efforts, adapted to prepare the pupil for the study of mental Arithmetic, and designed to be introductory to higher treatises on Algebra."

There is no class of Works in which the public are more deeply interested than in School Books, and when good ones are published, the author should be encouraged, and receive the commendation that his labors deserve. It is with this feeling that we always notice school books, and in the present instance we are happy in being able to speak favorably of a valuable addition to our stock of books, on a most interesting and important study, which, by means of this treatise, may be introduced with the greatest advantage into our public schools. We will only add, that the plan of the author is admirably executed.

The able Editor of the *Christian Reflector*, who was selected from the Boston School Committee to examine the Mathematical Department of their Schools, and who has just completed that arduous task, says of Tower's "*Intellectual Algebra*"—

"This is a new text-book, on a new plan, which we greatly admire. It is to the Algebraic science very much such a work as was Colburn's '*First Arithmetic*' to the science of common numbers. We observe that it is commended by experienced teachers. We shall certainly favor its adoption in the Mathematical department of the Schools of Boston, and recommend it to the attention of School Committees throughout the country."

The following is from the Principal of the celebrated Private School in Roxbury, one of the best in this country.

David B. Tower, Esq.,—Dear Sir: I have examined your "*Intellectual Algebra*" with some care and attention, and am much pleased with the plan and execution of the work. I think it admirably adapted for the early training of youthful minds in mathematics. I shall introduce it forthwith into my school.

Very truly and sincerely yours,

DANIEL LEACH.

Roxbury, August, 6, 1845.

From E. G. Storke, Esq., County Superintendent of Cayuga County

AUBURN, SEPT. 20, 1845.

Messrs Paine & Burgess,—The examination of "Towers' Intellectual Algebra" led me to remark that it was a work which I could cheerfully and heartily recommend, for its intrinsic value and excellence; and I avail myself of the first opportunity of doing so.

I regard it as the legitimate successor of Colburn's First Lessons, and it will, in my opinion, prove as valuable to the student of Algebra as that has been to the student of Arithmetic. It divests the science of its mystery and repulsiveness, and brings its *principles* clearly before the mental vision, so simplified and illustrated, that they can be readily comprehended by most pupils of from ten to twelve years of age.

I therefore hail with pleasure, this new and valuable incentive to *mental exercise* in our Schools, and am satisfied that the work has but to be examined to be approved and adopted. It is peculiarly adapted to the use of Common Schools, and to facilitate its introduction, we shall give the members of our Teachers' Institute, which is soon to convene, daily and thorough exercises in it.

Respectfully and truly Yours, E. G. STORKE.

BOSTON, SEPT. 23, 1845.

Dear Sir,—Having been absent from the city several months, I did not receive, so soon as I otherwise should, the copy of your book, the "Intellectual Algebra," which you did me the honor to send to my house. I have examined the book within a few days, and in my humble opinion, it is admirably adapted to the purposes for which it is intended.

It seems to me, you have very happily applied the "charms of logic" to that beautiful and much neglected study of Algebra, and if such a book could be freely introduced into our Common Schools I doubt not it would do more than almost anything else to invigorate and concentrate the intellectual powers of the young.

With much respect, your obliged servant,

JOHN T. SARGENT

DAVID B. TOWER, ESQ.

SALEM, JULY 26, 1845.

Mr. David B. Tower,—Dear Sir: It is thought by most Teachers at present, that children have not commenced the study of Arithmetic aright and radically, unless they have begun with "Colburn's First Lessons," or some other book of oral exercises. It appears to us that it is equally important that *Algebra* should be thus commenced. We rejoice to see a work of this kind from your hands; and the wonder is, that it has not entered the brain of some one before, to put one forth. Your "*Intellectual Algebra*," in our humble opinion, is a happy conception, and a design well executed,—leading the mind on by very easy and gradual steps, and by clear illustrations. We regard Algebra as an interesting and important study for children, and well calculated to aid their progress in common Arithmetic. We think, that if the merits of the study, and of your little book, are duly appreciated, it will be widely introduced into the Schools of our land.

Yours with esteem,

EDWIN JOCELYN,
Principal of F. High School.
CHARLES NORTHEND,
Principal of Epes School
D. P. GALLOUP,
Principal of Hacker School
A. C. SMITH,
Principal of Philip's School
J. B. FAIRFIELD,
Principal of Browne School

From Boston Recorder, July 31, 1845.

This work was prepared, the author informs us, for the use of the *blind* under his charge, and is now printed in hope that it may prove useful to the seeing. It is on the "inductive plan," and is believed to supply a deficiency in the books provided for young pupils. The operations are limited to small numbers, and lead the pupil on step by step towards higher mental efforts. The plan, and the execution of it, cannot fail to meet the approbation of Teachers.

BOSTON, SEPT., 15, 1845.

D. B. Tower, Esq.,—Dear Sir:—I have examined your "*Intellectual Algebra*," and cheerfully concur in the opinion expressed in the recommendation of the Principals of the Public Schools in Boston. Very respectfully yours,

R. W. WRIGHT,
Principal of the department of Mathematics in the Adams School

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